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Magnetism of Metals (金属磁性)

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Magnetism of Conduction electron 1 (伝導電子の磁性 1)

Pauli susceptibility (金属常磁性)

Magnetic Impurity in nonmagnetic material （磁性不純物）

Kondo effect 近藤効果

Interaction between atomic magnetic moments

Ferromagnetic or Antiferromagnetic? 強磁性・反強磁性?

Magnetic Alloy （磁性合金）

Magnetism of Conduction electron 2 (伝導電子の磁性 2)

Landau 反磁性, de Haas - van Alphen 効果

quantum Hall effect (2D)

（参考文献）

(1) **金森順次郎著「磁性」(培風館**, 1969)

(2) **斯波弘行著「基礎の固体物理学」（培風館**, 2007）

(3) **斯波弘行著「固体の電子論」（丸善**, 1996）

(4) J. R. Hook & H. E. Hall「Solid State Physics (2nd ed.)」(Wiley, 1991）

Many electron system with many nuclei: (Self-consistent calculation)

many Fermion system
strong Coulomb interaction $\Rightarrow$ Quasiparticles
weak interaction on (Ground state)
Crystal structure
Magnetic structure

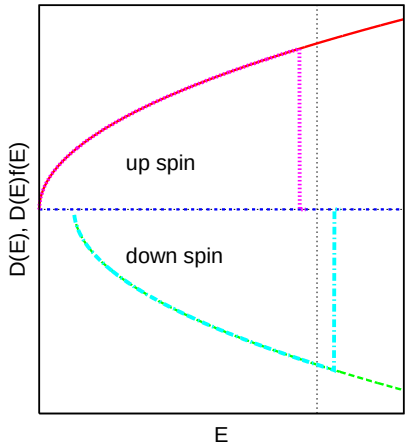
Fermi gas
Fermi liquid

(S.I.)

$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

1 Paramagnetism in nonmagnetic metal (**金属常磁性**)

Zeeman splitting : $(\pm\mu_{\rm B}B)$



$D(\epsilon)$: Density of states per spin

$M = \mu_{\rm B}B \times D(\epsilon_{\rm F}) \times 2 \times \mu_{\rm B}$

Paramagnetic susceptibility

in the noninteracting electrons : $(B \approx \mu_0 H)$

$$\chi_0 = \frac{M}{H} \approx \frac{\mu_0 M}{B} = 2\mu_0\mu_{\rm B}^2 D(\epsilon_{\rm F})$$

χ_0 can be obtained by the band structure calculation.

The susceptibility is enhanced, for example,

$$\chi_{\rm P} = \frac{\chi_0}{1 - I\chi_0}$$

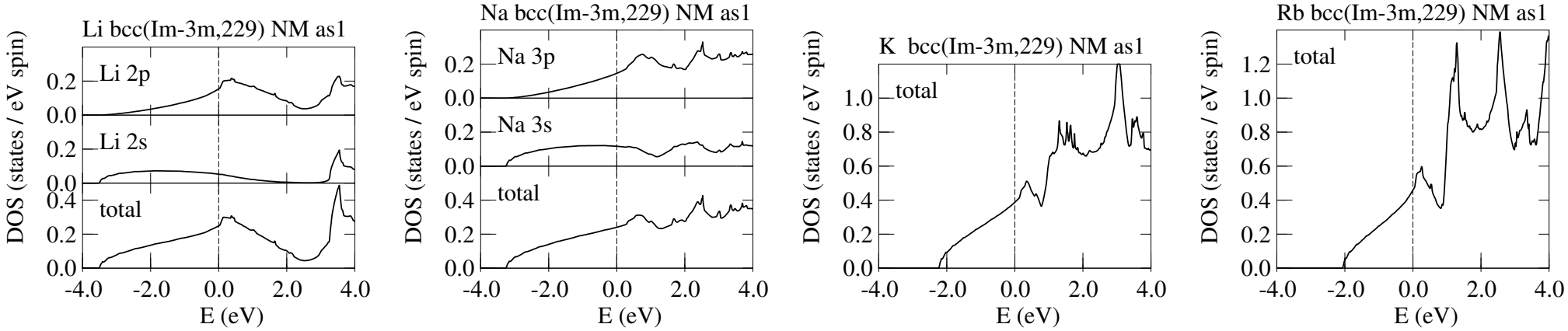
due to the electron-electron interaction.

	$\chi/10^{-5}$		
Metal	Free el.	Band cal.	exp.
	χ_0	χ_0	$\chi_{\rm P}$
Li	1.01	1.65	2.5
Na	0.83	0.86	1.4
K	0.67	0.72	1.1
Rb	0.63	0.70	1.0
Ti		3.8	18.1
V		9.9	34.6

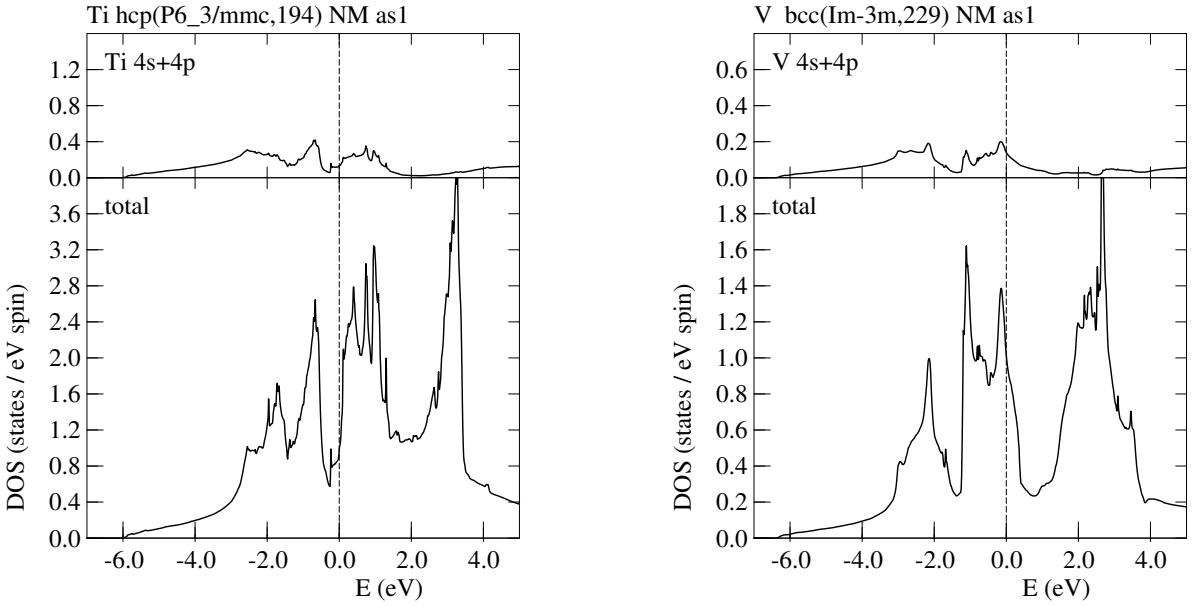
The temperature-independent susceptibility is treated as the Pauli susceptibility.

DOS [states/eV spin cell] given by the band structure calculation

Alkali metals



3d-Transition metals

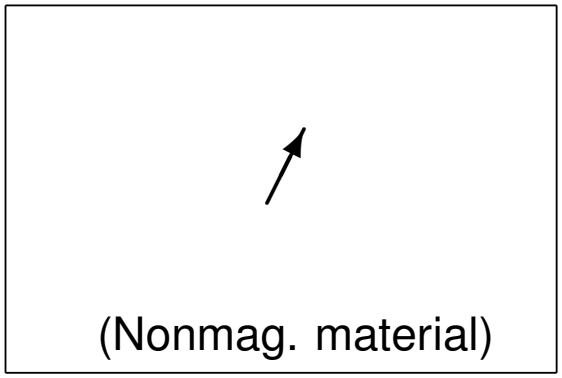


metal	DOS[au]
Li	0.099
Na	0.056
K	0.043
Rb	0.042
Ti	0.225
V	0.591

DOS=2$D(\epsilon_{\mathrm{F}})$

au = /Ha Bohr^3

2 Magnetic Impurity in Nonmagnetic Material



(Example) Fe in Mo

2.1 Anderson Model

Anderson Hamiltonian: ($n_\sigma = d_\sigma^\dagger d_\sigma$)

$$H = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}}\, c_{\boldsymbol{k}\sigma}^\dagger c_{\boldsymbol{k}\sigma} + \sum_\sigma \epsilon_{\mathrm{d}} n_\sigma + U n_\uparrow n_\downarrow + \frac{1}{\sqrt{N_{\mathrm{A}}}} \sum_{\boldsymbol{k}\sigma} \left\{ V_{\boldsymbol{k}}\, c_{\boldsymbol{k}\sigma}^\dagger d_\sigma + V_{\boldsymbol{k}}^*\, d_\sigma^\dagger c_{\boldsymbol{k}\sigma} \right\}$$

(N_{A}: the number of unit cells in the crystal)

Is the system "magnetic" or "nonmagnetic"?

2.1.1 Hartree-Fock approximation

($\sigma = \uparrow,\ \downarrow$)

$$H_\sigma = \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}} c_{\boldsymbol{k}\sigma}^\dagger c_{\boldsymbol{k}\sigma} + \epsilon_{\mathrm{d}} n_\sigma + U \langle n_{-\sigma} \rangle n_\sigma + \frac{1}{\sqrt{N_{\mathrm{A}}}} \sum_{\boldsymbol{k}} \left\{ V_{\boldsymbol{k}} c_{\boldsymbol{k}\sigma}^\dagger d_\sigma + V_{\boldsymbol{k}}^* d_\sigma^\dagger c_{\boldsymbol{k}\sigma} \right\}$$

Self-consistent calculation for $\langle n_\uparrow \rangle$ and $\langle n_\downarrow \rangle$

Energy : $E = \sum_\sigma \langle H_\sigma \rangle - U \langle n_\uparrow \rangle \langle n_\downarrow \rangle$

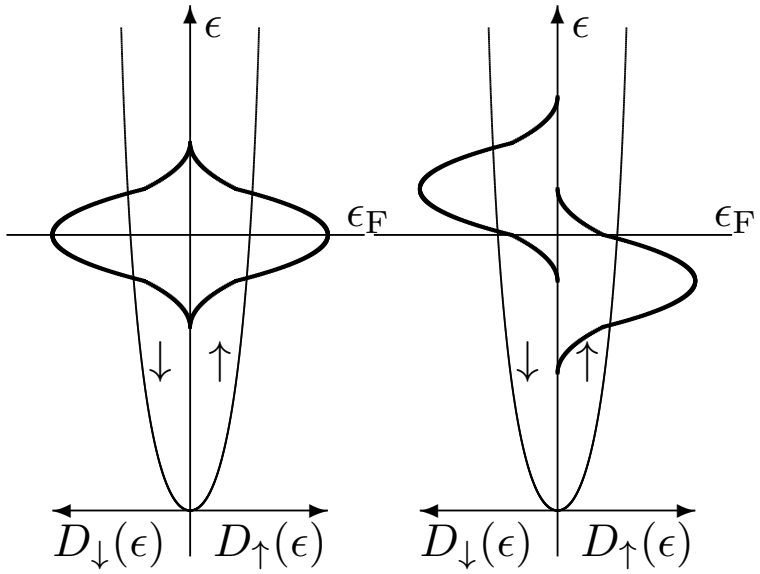
Virtual bound state

Partial DOS of d state:

$$D_{\mathrm{d}\sigma}(\epsilon) = \frac{\Delta/\pi}{(\epsilon - \epsilon_{\mathrm{d}\sigma})^2 + \Delta^2}$$

$$\epsilon_{\mathrm{d}\sigma} = \epsilon_{\mathrm{d}} + U \langle n_{-\sigma} \rangle$$

$$\Delta \approx \pi \left\langle |V_{\boldsymbol{k}}|^2 \right\rangle D_{\mathrm{c}}(\epsilon_{\mathrm{F}})$$



No moment Local moment

Virtual bound state

(parameters: $E_{\rm d}$, U, Δ)

Starting with no magnetic moment,

$$D_{\rm d}(\epsilon_{\rm F}) \equiv D_{\rm d\uparrow}(\epsilon_{\rm F}) = D_{\rm d\downarrow}(\epsilon_{\rm F})$$

Appearance of local moment:

$$UD_{\rm d}(\epsilon_{\rm F}) > 1$$

2.2 Local magnetic moment μ

Susceptibility has a Curie term:

$$\chi(T) = \chi_{\rm P} + \frac{B}{T}; \quad B = \frac{N\mu^2}{3k_{\rm B}}$$

example) Fe impurity in various 4d metals,

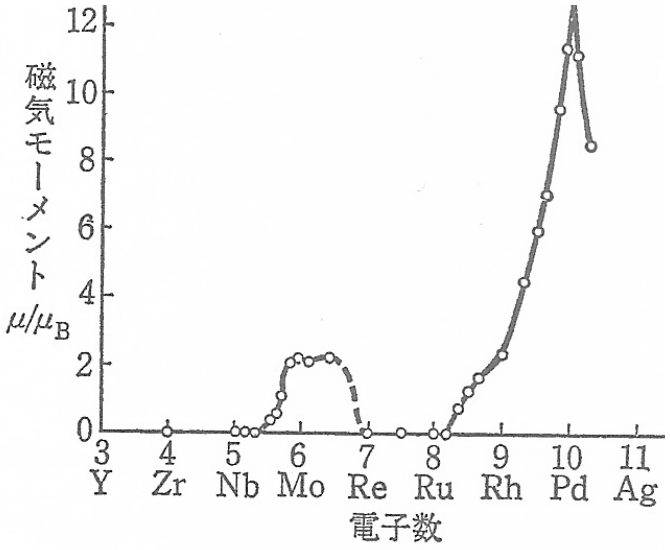
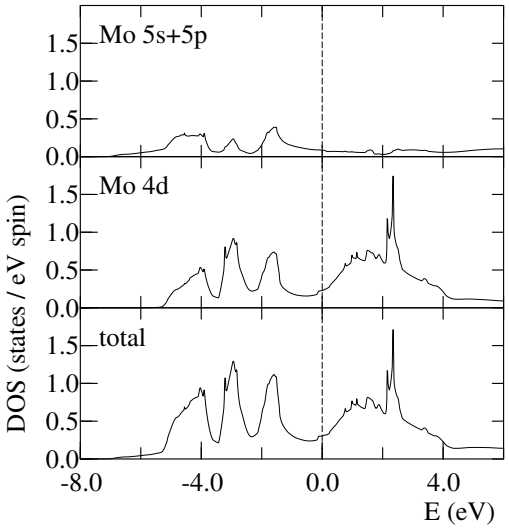


図 11-11 1% Fe を含む合金での Fe の
気モーメント (Clogston et al)

The Fe magnetic moment induces large moment at surrounding Pd atoms.

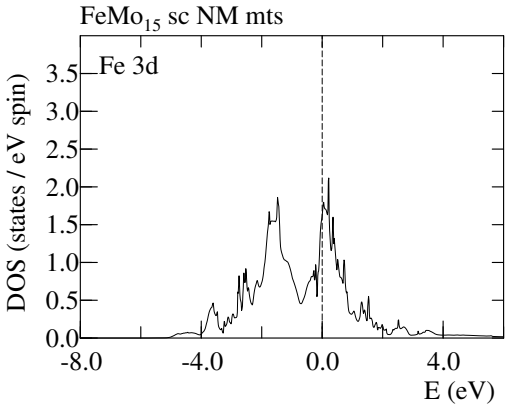
2.2.1 Band structure calculation: Fe in Mo (bcc*8-sc supercell)

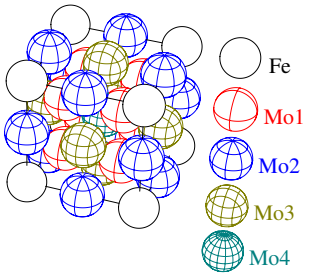
Mo



$FeMo_{15}$ NM

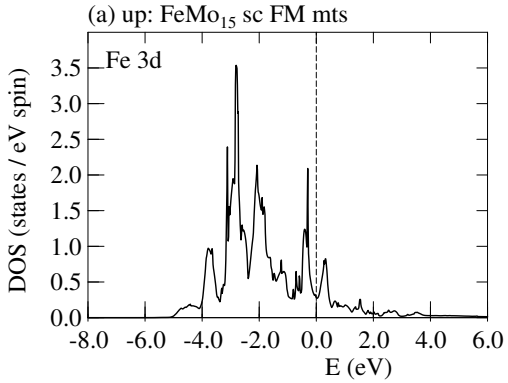
PDOS at Fe site

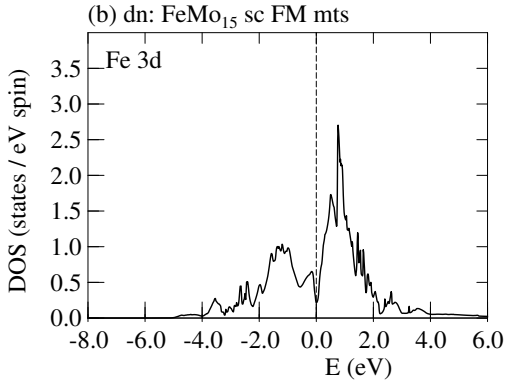




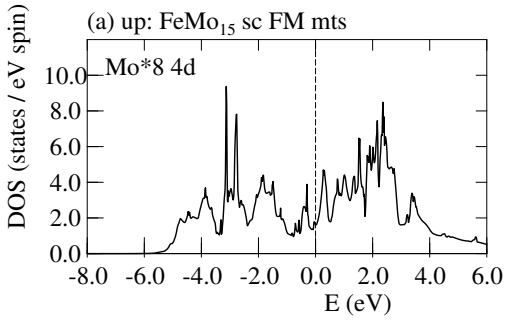
$FeMo_{15}$ FM

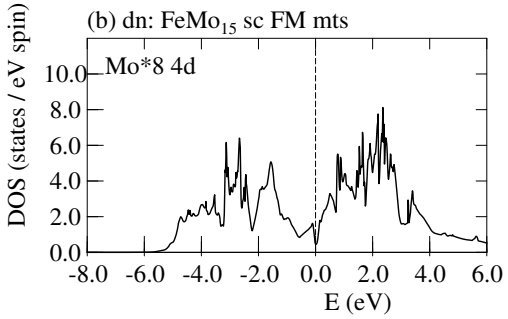
PDOS at Fe site





PDOS at Mo1 site



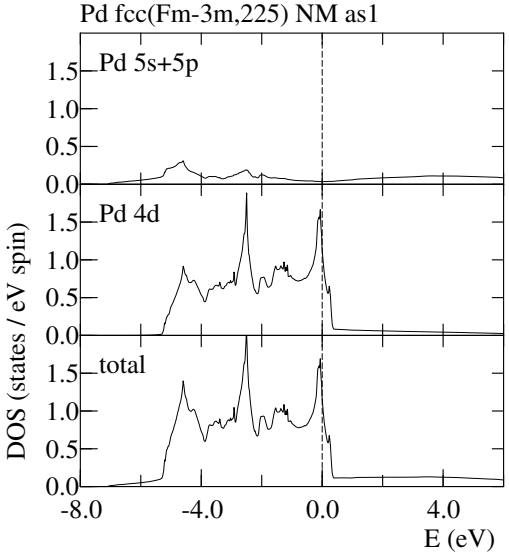


Magnetic moment [μ_B]

Total	Fe	Mo1	Mo2
2.08	2.12	-0.01	0.02

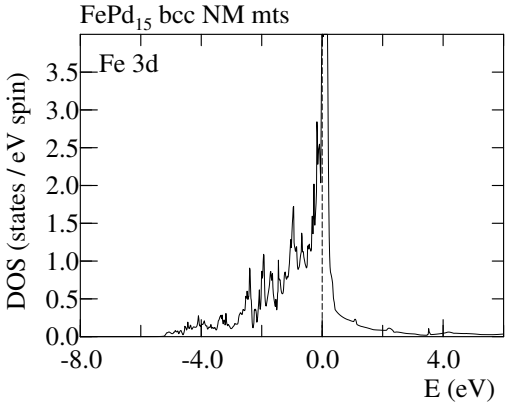
2.2.2 Band structure calculation: Fe in Pd (fcc*8-bcc supercell

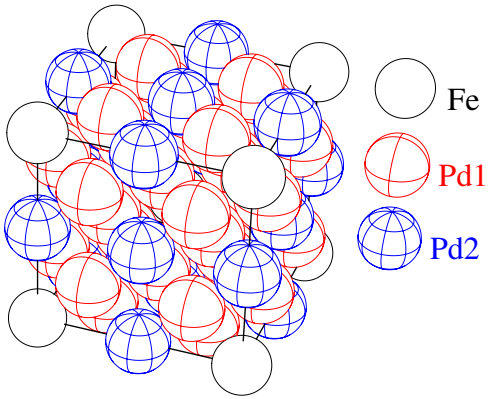
Pd



$FePd_{15}$ NM

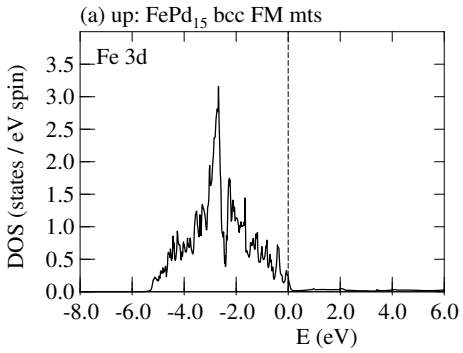
PDOS at Fe site

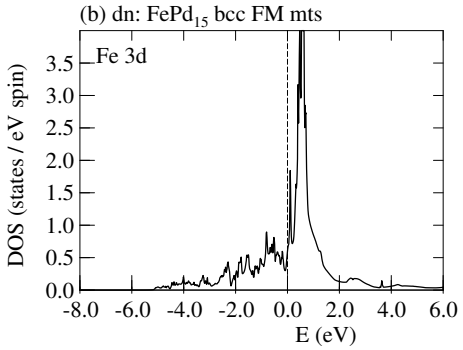




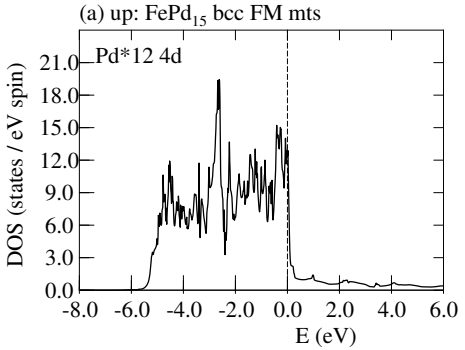
$FePd_{15}$ FM

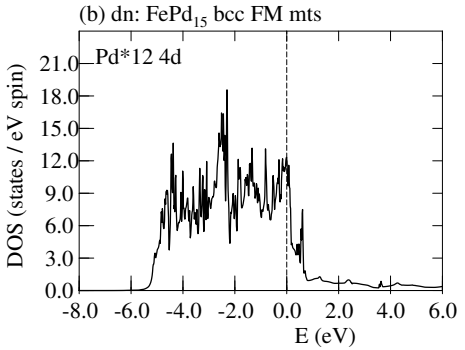
PDOS at Fe site





PDOS at Pd1 site





Magnetic moment [μ_B]

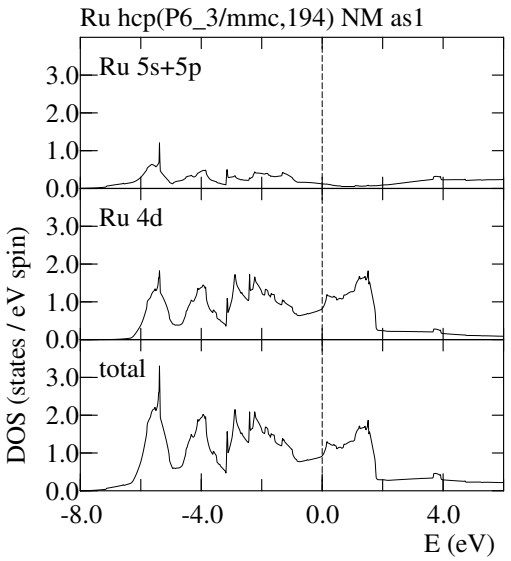
Total	Fe	Pd1	Pd2
6.69	3.19	0.22	0.24

Large moments are induced at many Pd sites.

The supercell is too small.

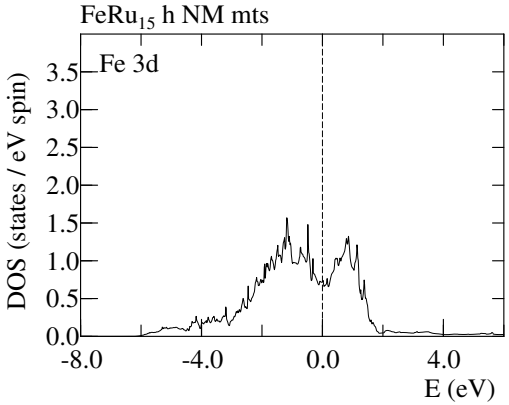
2.2.3 Band structure calculation: Fe in Ru (hcp*8-h supercell)

Ru



$FeRu_{15}$ NM

PDOS at Fe site



$FeRu_{15}$ FM

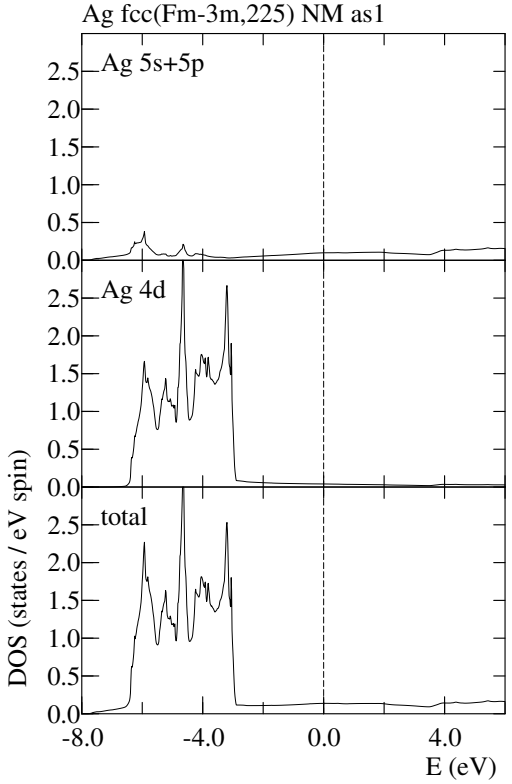
No magnetic solution

The Fe impurity in Ru is really nonmagnetic.

The LDA/GGA calculation works well.

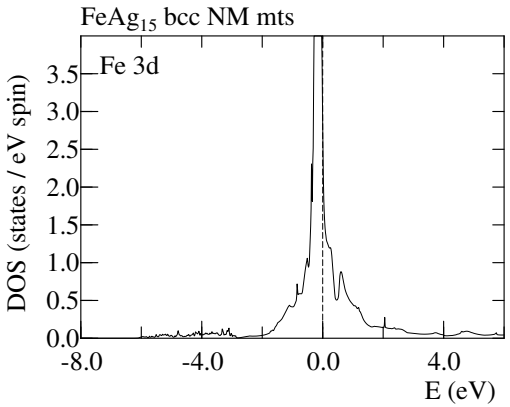
2.2.4 Band structure calculation: Fe in Ag (fcc*8-bcc supercell)

Ag



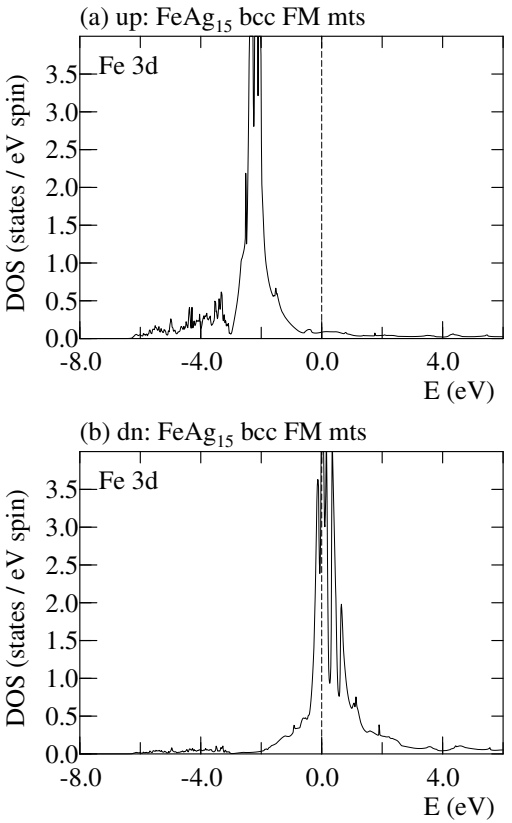
$FeAg_{15}$ NM

PDOS at Fe site



$FeAg_{15}$ FM

PDOS at Fe site



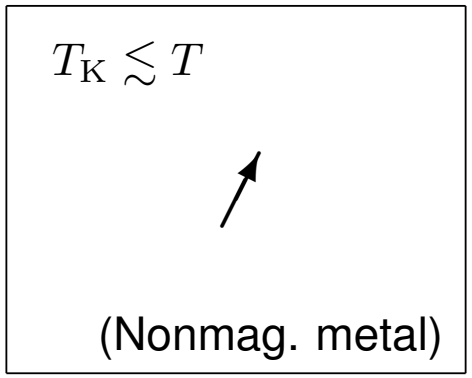
The LDA/GGA calculation has a magnetic solution.

The LDA/GGA calculation fails to predict the ground state.

At low temperatures $T \lesssim T_{\mathrm{K}}$, the local moment disappears.

Kondo effect

3 Magnetic Impurity in nonmganetic metal (Kondo effect: $T \lesssim T_{\rm K}$)



(Example) Fe in Cu

At high temperatures

Susceptibility : Curie low

Local magnetic moment

$\Downarrow$

At low temperatures

Susceptibility : $\rightarrow$ a constant

No local moment

Kondo singlet state

Kondo effect

3.1 Anderson Model

Anderson Hamiltonian: ($n_\sigma = d^\dagger_\sigma d_\sigma$)

$$H = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}\sigma}\, c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \sum_\sigma \epsilon_{\rm d} n_\sigma + U n_\uparrow n_\downarrow$$
$$+ \frac{1}{\sqrt{N_{\rm A}}} \sum_{\boldsymbol{k}\sigma} \left\{ V_{\boldsymbol{k}}\, c^\dagger_{\boldsymbol{k}\sigma} d_\sigma + V^*_{\boldsymbol{k}}\, d^\dagger_\sigma c_{\boldsymbol{k}\sigma} \right\}$$

For a "magnetic" impurity:

$V_{\boldsymbol{k}} \ll U$

$\Downarrow$

Large U perturbation theory breaks down

at low temperatures.

($T \lesssim T_{\rm K}$)

3.2 Large-U perturbation theory

$|F\rangle$: Free electron ground state ('Fermi sea')

Unperturbed ground states:

$$d_{\downarrow}^{\dagger}|F\rangle\,, \quad d_{\uparrow}^{\dagger}|F\rangle$$

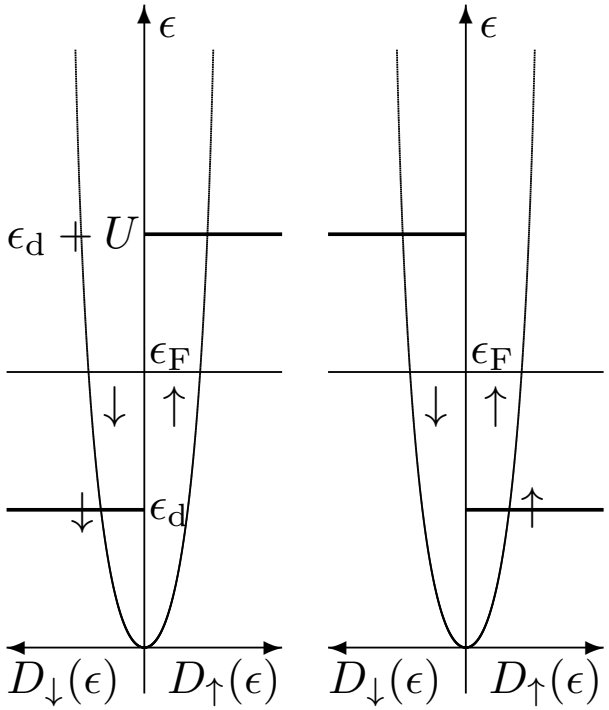
Perturbation term in Anderson Hamiltonian

$$H' = \frac{1}{\sqrt{N_{\mathrm{A}}}} \sum_{\boldsymbol{k}\sigma} \left\{ V_{\boldsymbol{k}} c_{\boldsymbol{k}\sigma}^{\dagger} d_{\sigma} + V_{\boldsymbol{k}}^{*} d_{\sigma}^{\dagger} c_{\boldsymbol{k}\sigma} \right\}$$

Approximations:

$$\boxed{\begin{array}{l} V_{\boldsymbol{k}\sigma} = V \quad (\ll U) \\ \epsilon_{\boldsymbol{k}\sigma} \sim \epsilon_{\mathrm{F}} = 0 \end{array}}$$

$\Downarrow$ (2-nd order perturbation)



Unperturbed system

Effective Hamiltonian:

$$\begin{aligned} H_{\mathrm{eff}} =& \frac{1}{N_{\mathrm{A}}} \sum_{\boldsymbol{k}\boldsymbol{k}'\sigma} \frac{V^2}{2} \left(\frac{1}{-\epsilon_{\mathrm{d}}} - \frac{1}{\epsilon_{\mathrm{d}} + U} \right) {c_{\boldsymbol{k}\sigma}}^{\dagger} c_{\boldsymbol{k}'\sigma} \\ &+ \frac{1}{N_{\mathrm{A}}} \sum_{\boldsymbol{k}\boldsymbol{k}'\sigma\sigma'} \frac{V^2}{2} \left(\frac{1}{-\epsilon_{\mathrm{d}}} + \frac{1}{\epsilon_{\mathrm{d}} + U} \right) {c_{\boldsymbol{k}\sigma}}^{\dagger} \boldsymbol{\sigma}_{\sigma\sigma'} c_{\boldsymbol{k}'\sigma'} \cdot \boldsymbol{S} \end{aligned}$$

$\epsilon_{\mathrm{d}} < 0, \quad -\epsilon_{\mathrm{d}} \sim \epsilon_{\mathrm{d}} + U > 0$

The first term $= 0$, when $-\epsilon_{\rm d} = \epsilon_{\rm d} + U$,

The second term:

(spin-flip scattering of conduction electron)

$$H_{\rm K} = -\frac{J}{2N_{\rm A}} \sum_{\boldsymbol{k}\boldsymbol{k}'\sigma\sigma'} {c_{\boldsymbol{k}\sigma}}^{\dagger} \boldsymbol{\sigma}_{\sigma\sigma'} c_{\boldsymbol{k}'\sigma'} \cdot \boldsymbol{S}$$

$$J = -2V^2 \left(\frac{1}{-\epsilon_{\rm d}} + \frac{1}{\epsilon_{\rm d} + U} \right) = -\frac{8V^2}{U} < 0$$

Kondo model

Antiferromagnetic exchange interaction

Local spin $\boldsymbol{S}$ attracts antiparallel conduction spin $\boldsymbol{\sigma}$.

$\Rightarrow$ Kondo singlet at low temperatures ($T \lesssim T_{\rm K}$)

Kondo temperature : $T_{\rm K}$

Scaling (renormarization of J) theory gives

$$k_{\rm B} T_{\rm K} \sim W \exp \frac{1}{J D_{\rm c}(\epsilon_{\rm F})} = W \exp \frac{-\pi U}{8\Delta}$$

where W is the conduction band width and $\Delta = \pi |V|^2 D_{\rm c}(\epsilon_{\rm F})$ is the width of virtual bound state in the Anderson model.

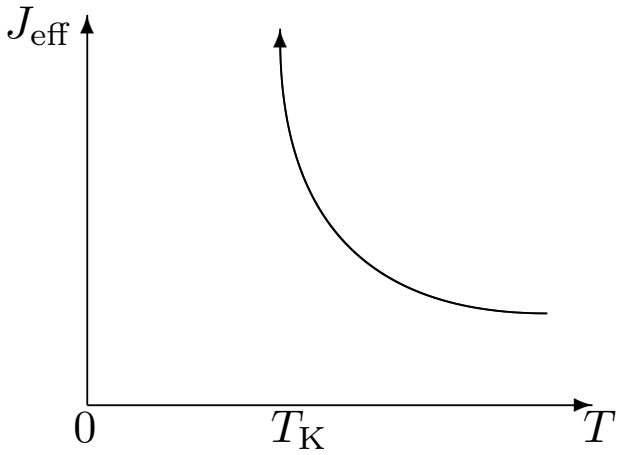
Scaling theory:

$|J_{\rm eff}| \to$ larger $\quad (T \to$ lower)

$$J_{\rm eff} \to -\infty \quad (T \to T_{\rm K} + 0)$$

Kondo singlet state is created.

The large U perturbation expansion breaks down in $T \lesssim T_{\rm K}$.



3.3 Perturbation theory from $U = 0$

Unperturbed system: $U = 0$

No local magnetic moment

"Fermi liquid"

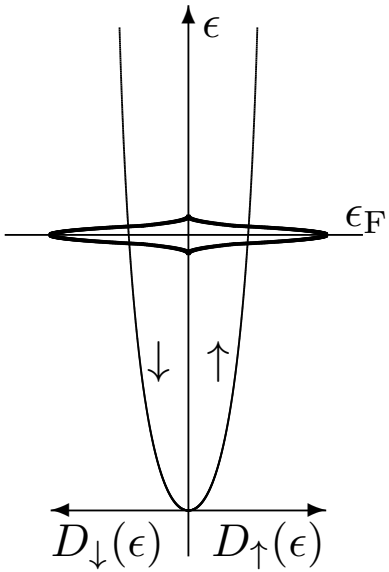
Perturbation: $H' = U n_\uparrow n_\downarrow$

$\downarrow$ (even if $U \to \infty$)

No local moment state is kept.

"Kondo singlet"

This has the property of the "Fermi liquid".



No moment

DFT must describe this "Fermi liquid" state (ground state).

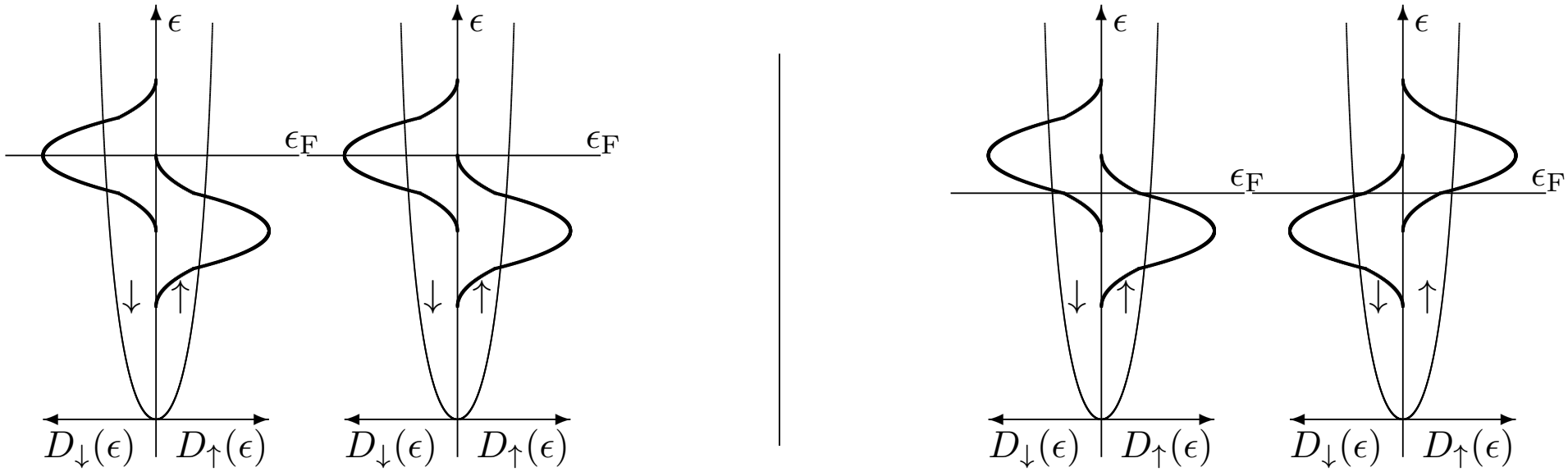
But usual approximation (LDA, GGA, ...) may not express the state appropriately.

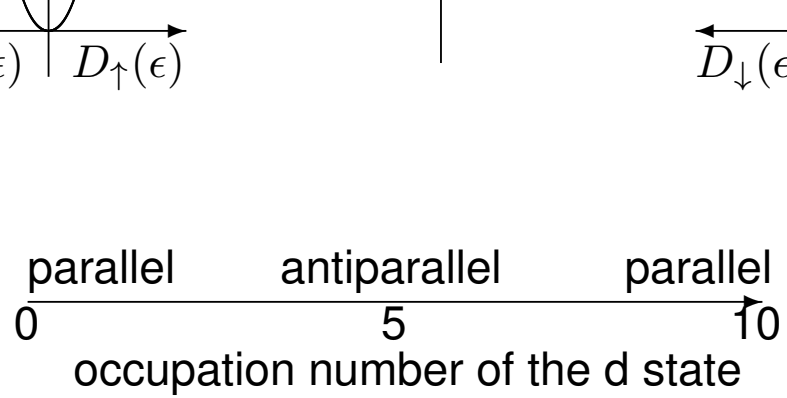
$$E(\text{Kondo}) < E(\text{M}) << E(\text{NM}); \quad E(\text{Kondo}) - E(\text{M}) \approx k_{\rm B} T_{\rm K}$$

4 Magnetic Interaction between Atomic Moments

4.1 A pair of magnetic atoms

Two virtual bound states ($E_{\rm d}$, U, Δ) + Electron transfer (t) between those v. b. states





4.2 Ferromagnetic state v.s. Antiferromagnetic state in a crystal

Hubbard model

Hubbard Hamiltonian:

$$H = \sum_{i\sigma} \epsilon_i n_{i\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + \sum_{(i,j)\sigma} t_{ij} {a_{i\sigma}}^\dagger a_{j\sigma}$$

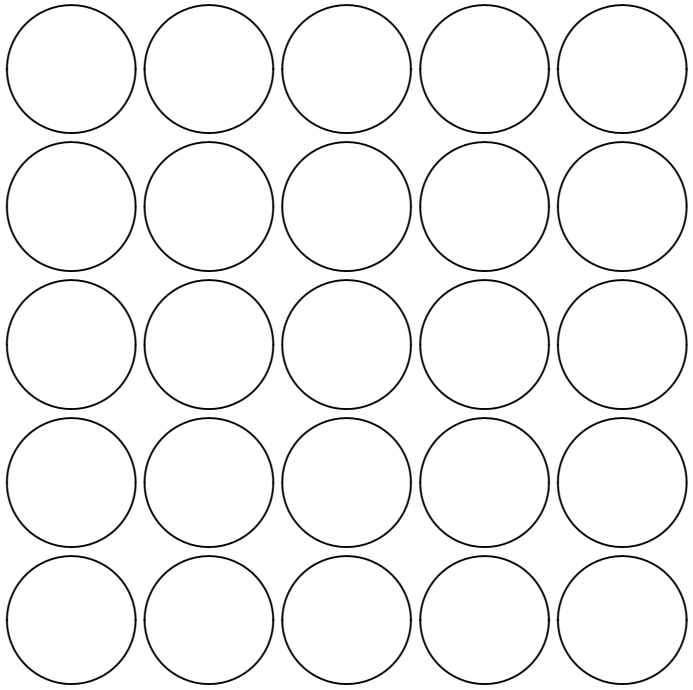
$$n_{i\sigma} = {a_{i\sigma}}^\dagger a_{i\sigma}$$

Hartree-Fock approximation:

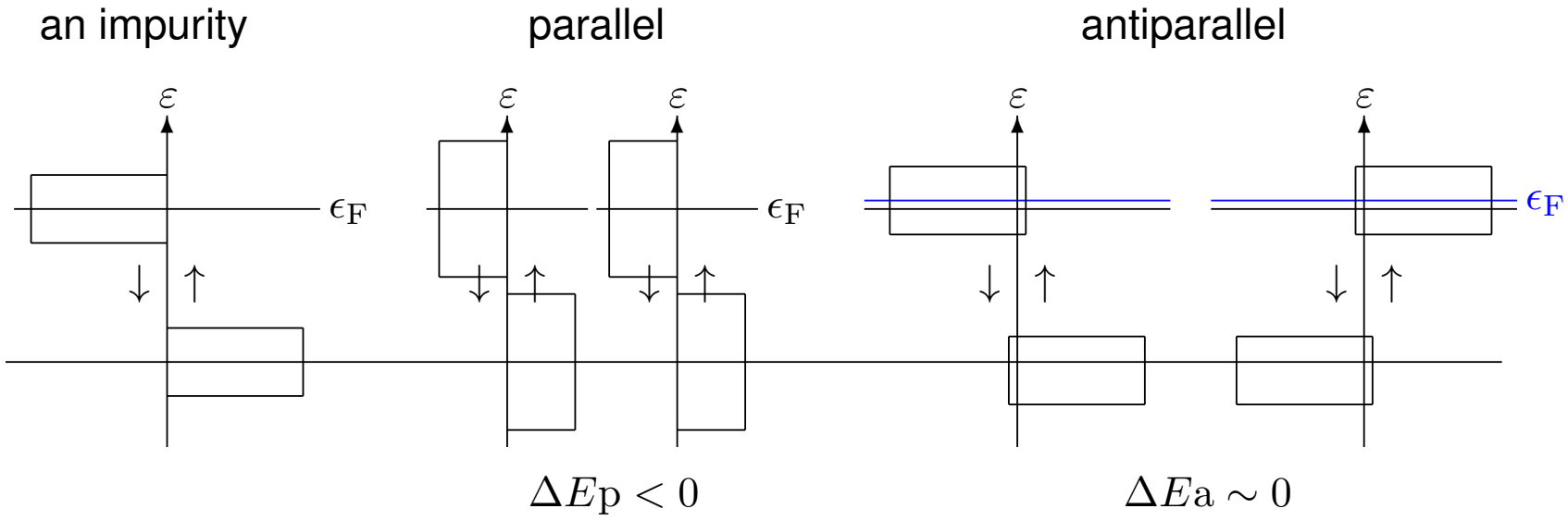
$$H_\sigma = \sum_{i\sigma} \epsilon_{i\sigma} n_{i\sigma} + \sum_{(i,j)\sigma} t_{ij} {a_{i\sigma}}^\dagger a_{j\sigma}$$

$$\epsilon_{i\sigma} = \epsilon_i + U_i \langle n_{i-\sigma} \rangle$$

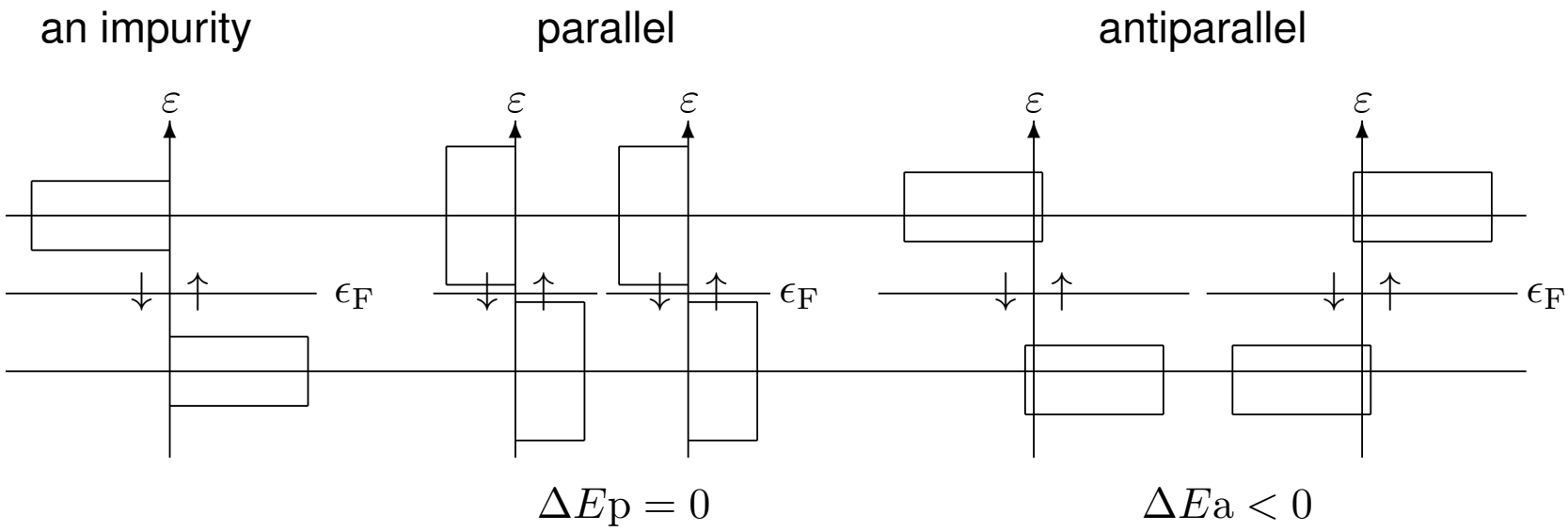
A s-orbital at a site



3 quater filled



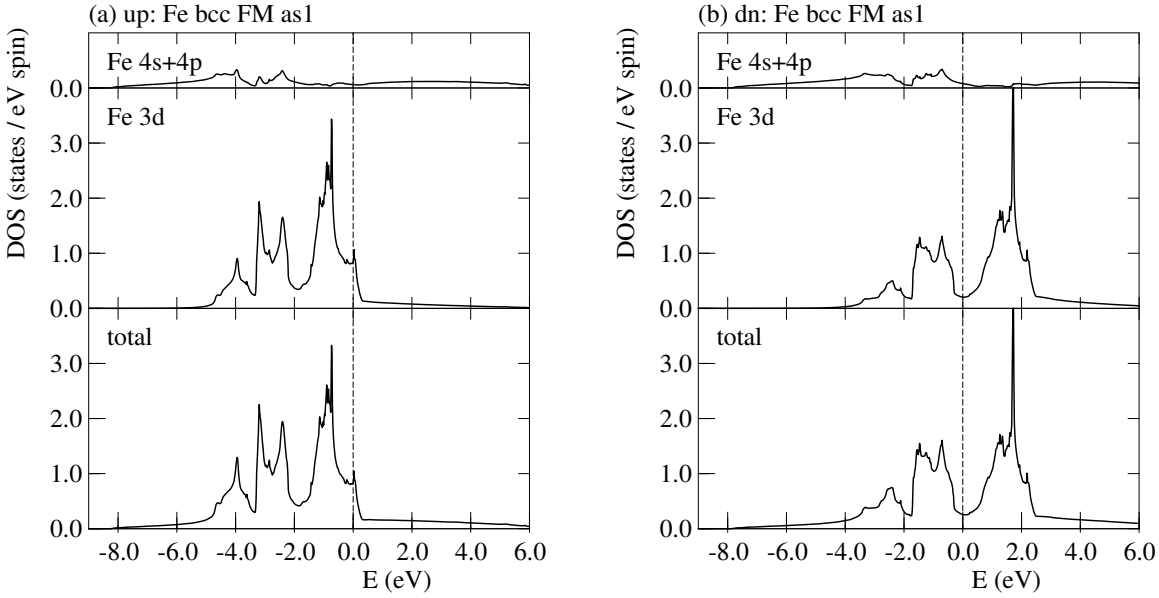
half filled



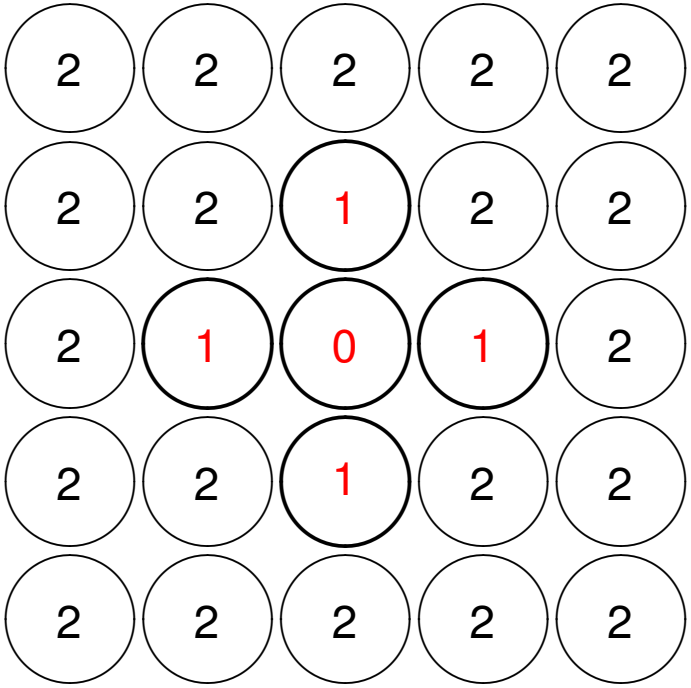
The covalent-bond concept is useful for understanding the magnetic interaction.

4.3 Examples of ferromagnetic metal

metal	crystal	$\mu\ [\mu_{\mathrm{B}}]$	T_{c}
Fe	bcc	2.22	1043
Co	hcp	1.72	1388
Ni	fcc	0.606	627
Gd	hcp	7.63	292
Dy	hcp	10.2	88



Nearest neighbor shell



0: the center, 1: the first neighbors

2: others

4.4 Nonlocal susceptibility

Example) 3d transition metals

Magnetic structure?

- ferromagnetic: Fe, Co, Ni
- antiferromagnetic: Cr, Mn
  (spin density wave, etc.)

Local susceptibility: $\chi_{00}(\epsilon_{\rm F}) = \dfrac{M_0}{H_0}$

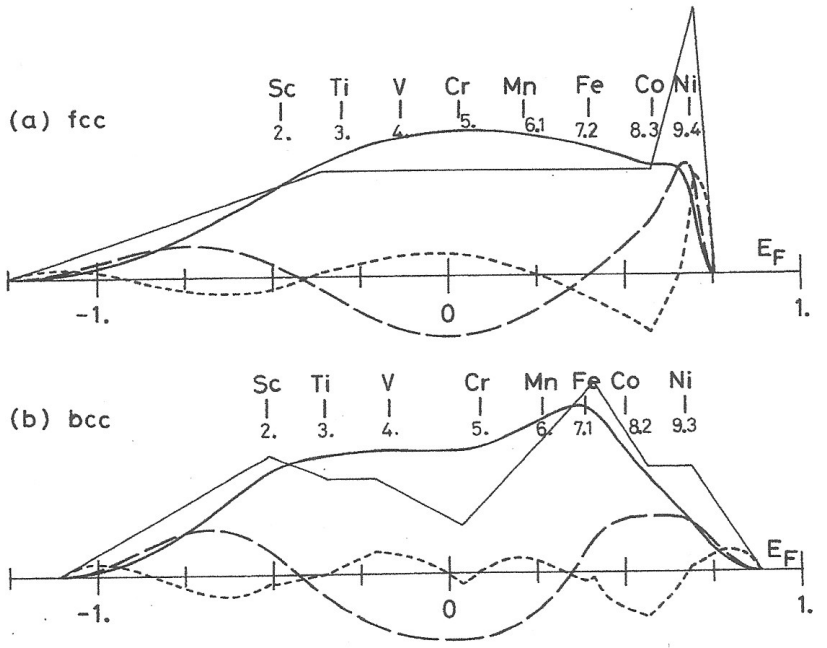
Nonlocal susceptibilities:

$$\chi_{01}(\epsilon_{\rm F}) = \frac{M_0}{H_1}, \quad \chi_{02}(\epsilon_{\rm F}) = \frac{M_0}{H_2}$$

PDOS at the center:

$$D_0(\epsilon_{\rm F}) = \chi_{00}(\epsilon_{\rm F}) + \chi_{01}(\epsilon_{\rm F}) + \chi_{02}(\epsilon_{\rm F})$$

Local and non-local susceptibilities



—— $D_0(\epsilon_{\rm F})$,

—— $\chi_{00}(\epsilon_{\rm F})$, – – – $\chi_{01}(\epsilon_{\rm F})$, ------ $\chi_{02}(\epsilon_{\rm F})$

$\chi_{01} > 0$ (FM) for $n_{\rm d} \lesssim 3$ or $n_{\rm d} \gtrsim 7$

$\chi_{01} < 0$ (AFM) for $3 \lesssim n_{\rm d} \lesssim 7$

The trend is similar for other lattices, although $n_{\rm d}$ is shifted.

5 Magnetic Alloys (磁性合金)

3d transition-metal alloys

Slater-Pauling curve

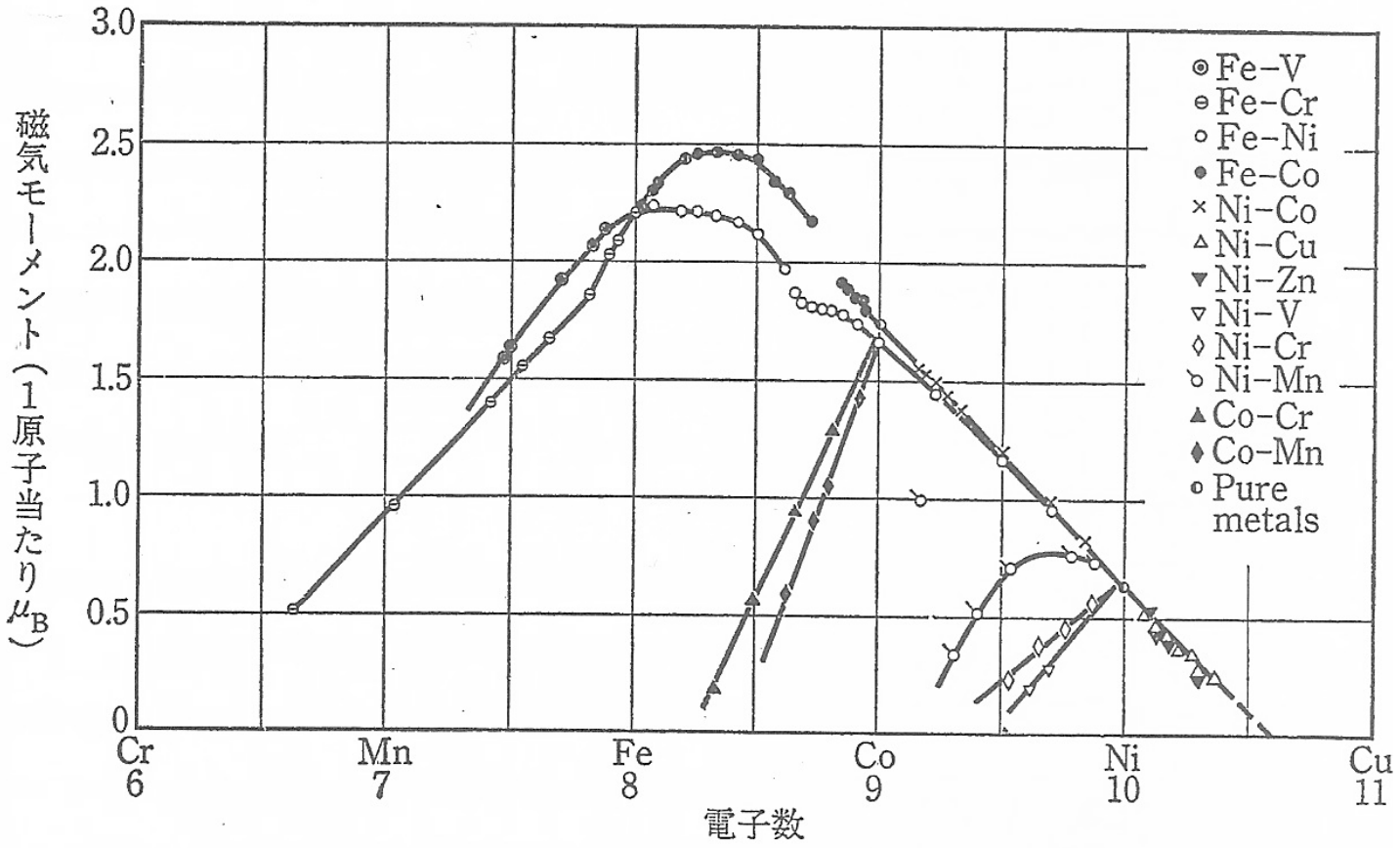


図 11-1 Slater-Pauling 曲線 (Bozorth)

5.1 Hubbard model

Hubbard Hamiltonian:

$$H = \sum_{i\sigma} \epsilon_i n_{i\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + \sum_{(i,j)\sigma} t_{ij} {a_{i\sigma}}^\dagger a_{j\sigma}$$

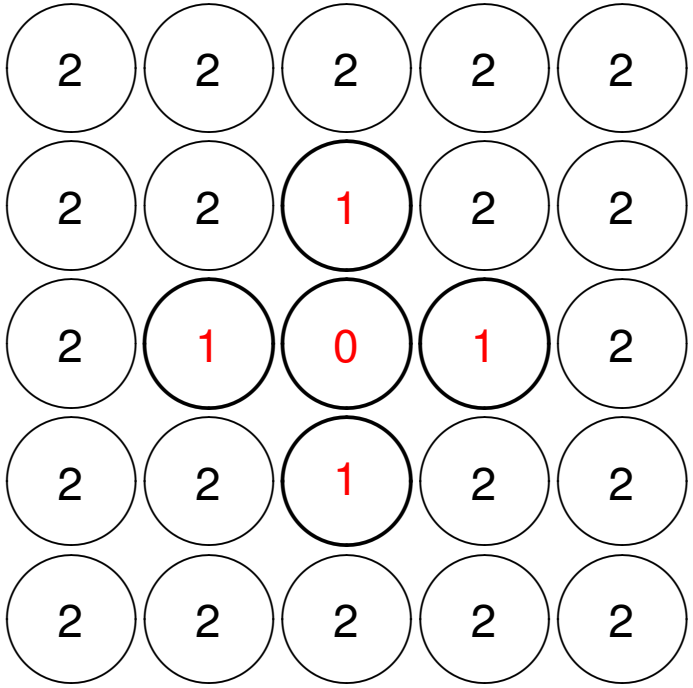
$$n_{i\sigma} = {a_{i\sigma}}^\dagger a_{i\sigma}$$

Hartree-Fock approximation:

$$H_\sigma = \sum_{i\sigma} \epsilon_{i\sigma} n_{i\sigma} + \sum_{(i,j)\sigma} t_{ij} {a_{i\sigma}}^\dagger a_{j\sigma}$$

$$\epsilon_{i\sigma} = \epsilon_i + U_i \langle n_{i-\sigma} \rangle$$

A s-orbital at a site



A cluster in metal

0: the center, 1: the first neighbors

2: others

5.2 3d transition-metal alloys

Cluster CPA — 2: coherent potential —

5.2.1 FeNi alloy (bcc, fcc)

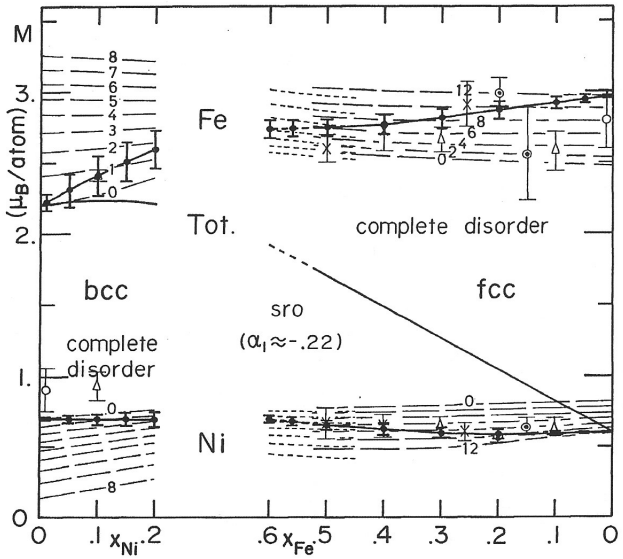
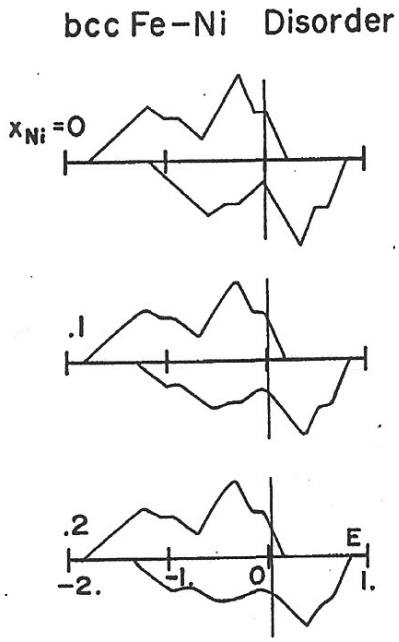
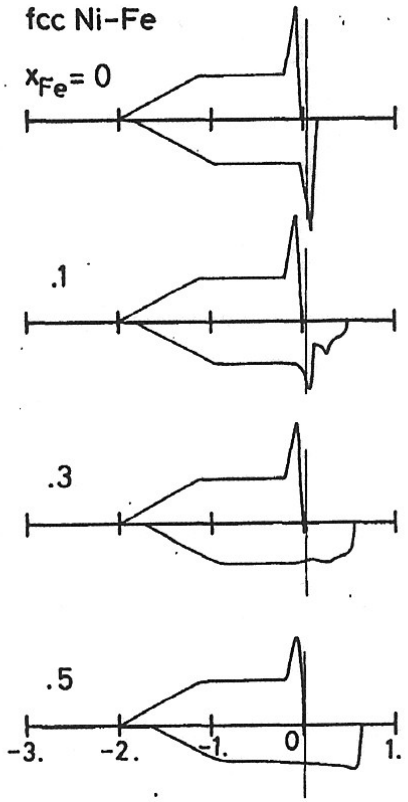


Fig. 4. The Ni and Fe atomic moments in fcc and bcc Ni–Fe. The dashed curves show the Ni and Fe moment for a given number of Ni nearest neighbors as is indicated in the figure. The full lines are the average Ni and Fe moment with the virtical bars representing the root mean square of the deviation of the moment from its average value. The curve labelled Tot. represents the total magnetization per atom. The dotted curves indicate the corresponding moments in alloys with atomic short-range order ($\alpha_1<0$). ×,[8] △,[9,10] ○[11] and ⊙[12] show the experimental data by neutron diffraction.





- Fe: $2.2\mu_{\mathrm{B}} \rightarrow \sim 3\mu_{\mathrm{B}}$
- Ni: $0.6\mu_{\mathrm{B}}$ constant

$\sim$

- Majority-spin band: fully filled?
- Charge neutrality

5.2.2 FeCo alloy (bcc)

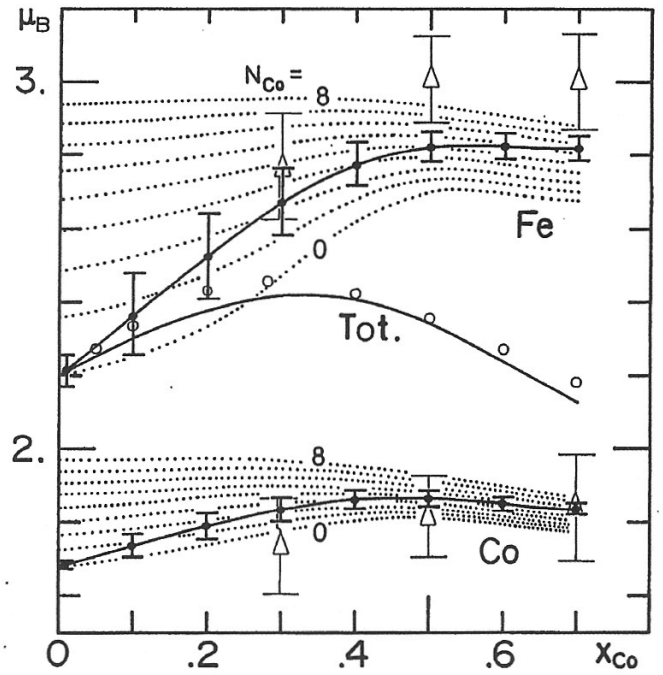


Fig. 12. The Fe and Co moments in bcc Fe–Co alloy with complete disorder. Triangles show the neutron diffuse scattering data for Fe and Co moments.[10] Circles display the experimental magnetization data.[13]

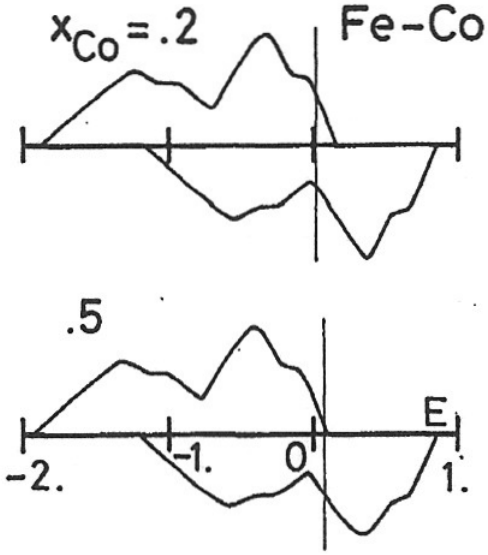


Fig. 9. The total DOS's in bcc Fe–Co.

- Fe: 2.2$\mu_{\mathrm{B}} \rightarrow \sim 3\mu_{\mathrm{B}}$
- Co: 1.8μ_{B} constant

$\sim$

- Majority-spin band: fully filled?
- Charge neutrality

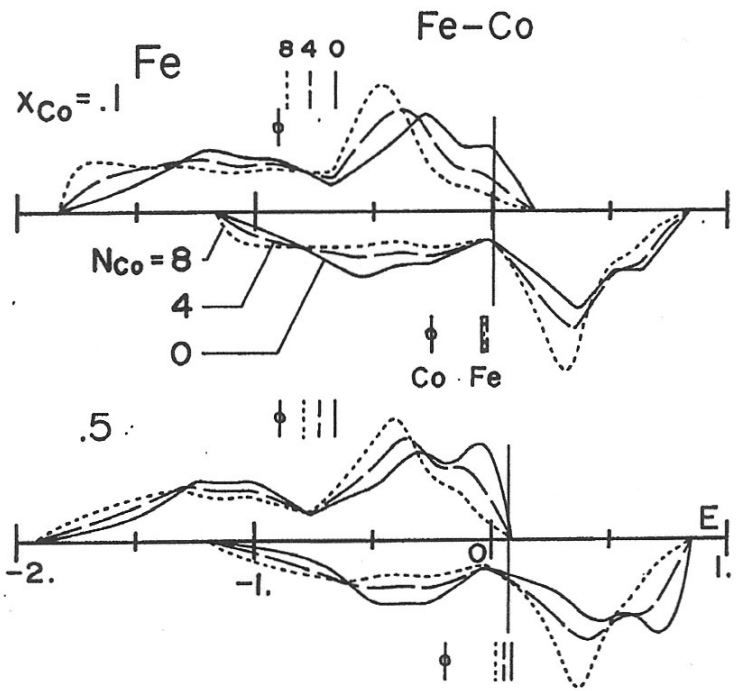


Fig. 10. The local DOS's and the atomic levels of Fe atom in bcc Fe–Co.

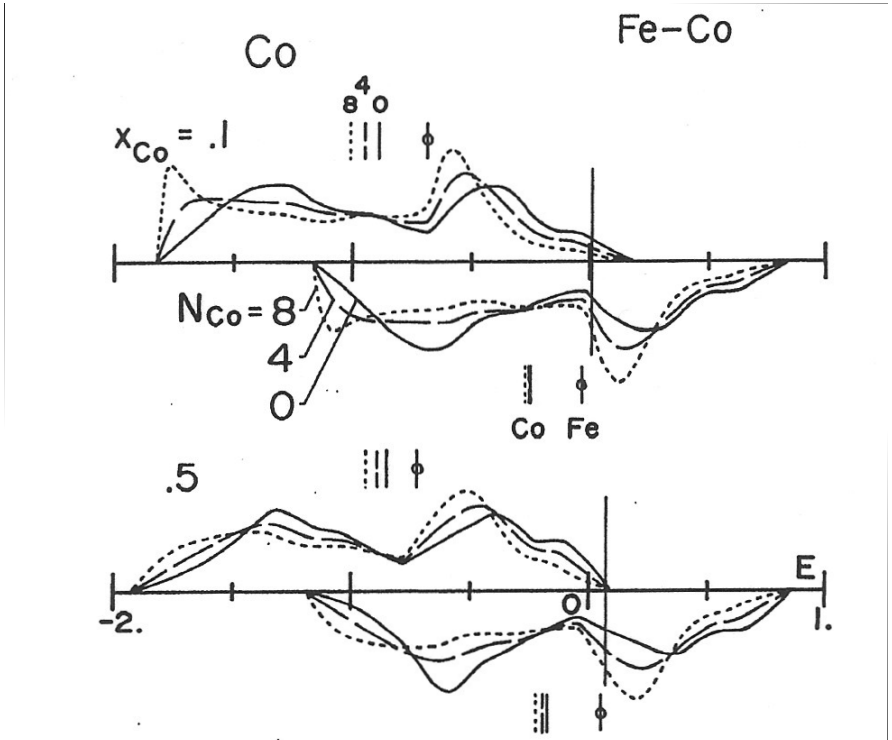


Fig. 11. The local DOS's and the atomic levels of Co atom in bcc Fe–Co.

Neighbor cobaltized Fe makes the Fe moment larger.

$\Rightarrow$ Large magnetization of NdFeB-magnet

5.2.3 VFe alloy (bcc)

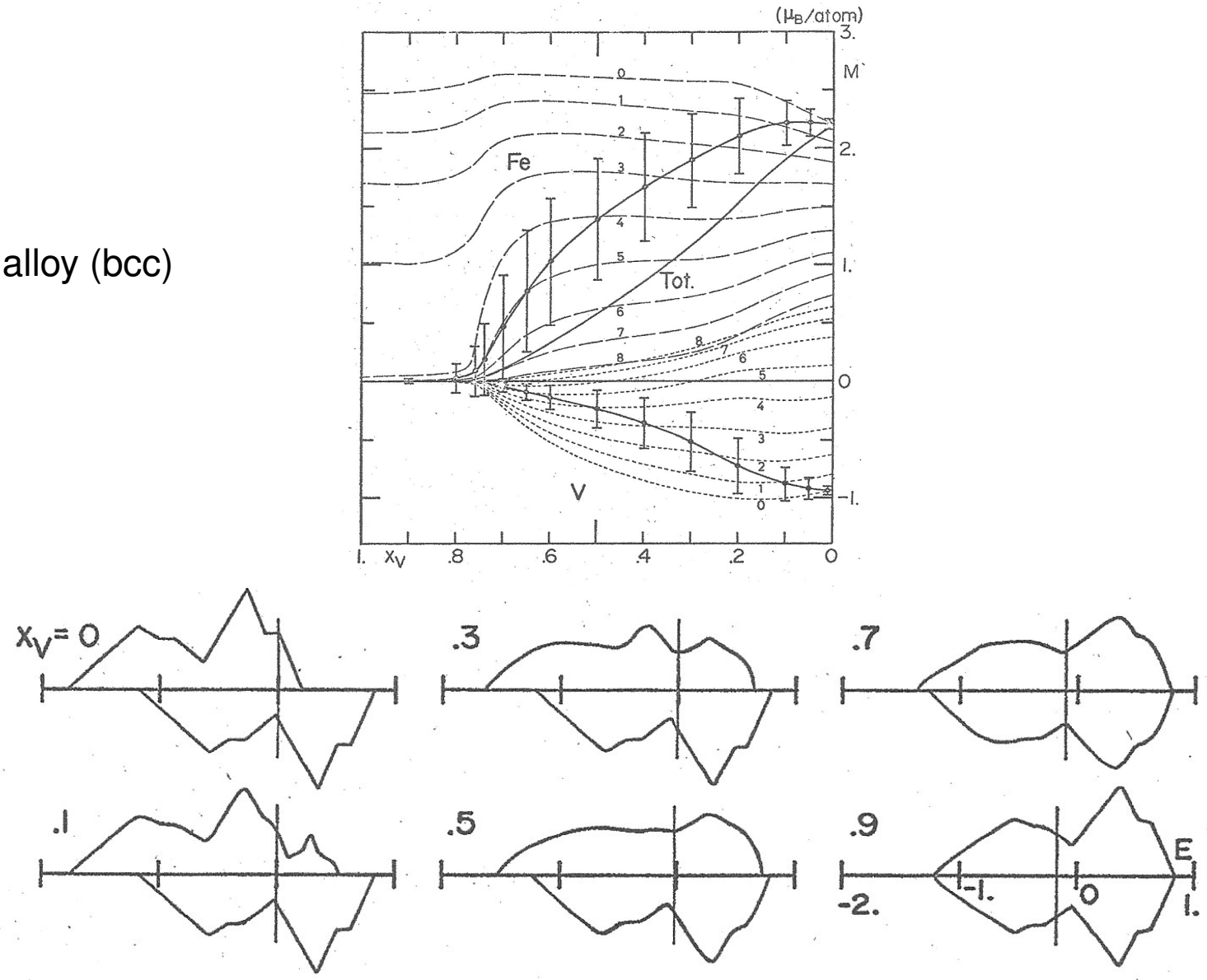
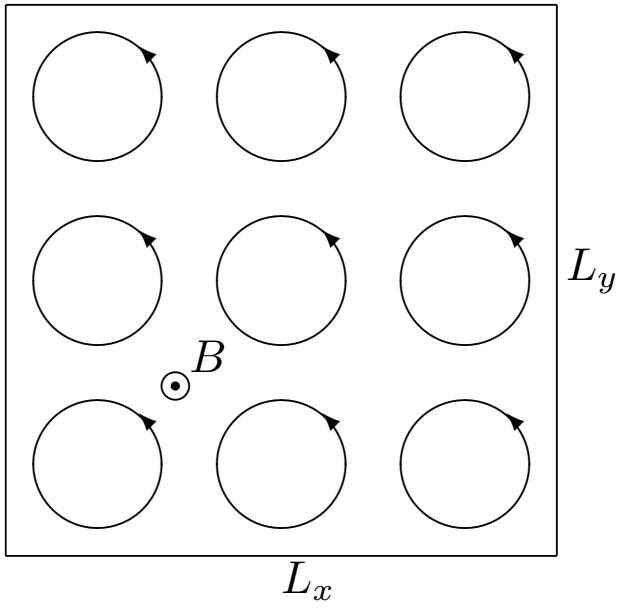


Fig. 3. The total density of states for each spin in various V concentrations. The vertical line near $E=0$ is the Fermi level.

6 Free electron under the uniform magnetic field



(Classical picture)

Eq. of motion: $m\dfrac{v^2}{r} = evB$

$\boxed{\omega_{\rm c} = \dfrac{v}{r} = \dfrac{eB}{m}}$: cyclotron frequency

v, r : arbitrary

When $B = 1$ T,

$\omega_{\rm c} \approx 1.8\times10^{11}$ rad·s^{-1}, $\hbar\omega_{\rm c} \approx 1.2$ meV

6.1 Landau level

(Quantum mechaical treatment)

Magnetic field: $\boldsymbol{B} = (0, 0, B) = \mathrm{rot}\boldsymbol{A}$

Vector potential: $\boldsymbol{A} = \left(-\dfrac{By}{2}, \dfrac{Bx}{2}, 0\right)$

Free electron system:

$$H = \frac{1}{2m}\left[\left(p_x - \frac{eBy}{2}\right)^2 + \left(p_y + \frac{eBx}{2}\right)^2 + p_z^2\right]$$

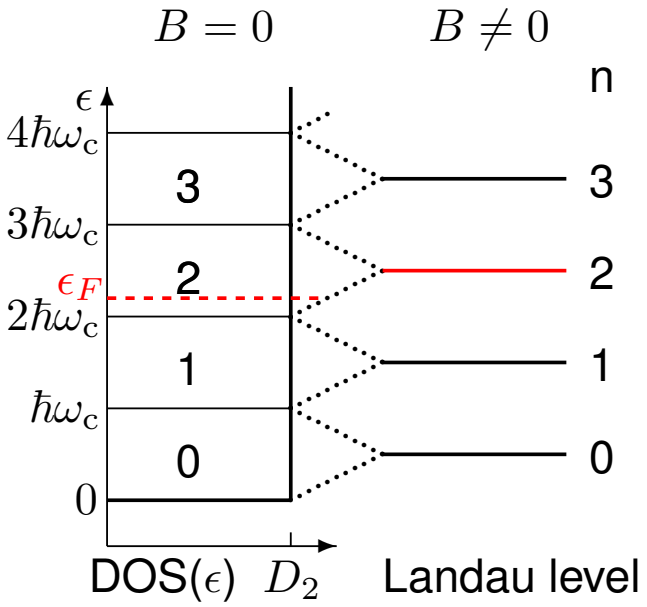
Eigenenergy:

$$\epsilon_n(k_z) = \hbar\omega_{\rm c}\left(n + \frac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m}$$

$$n = 0, 1, 2, \cdots, \quad k_z = \frac{2\pi}{L_z}n_z \ \ (n_z = \text{integer})$$

n: Landau level index

2-D free-electron system



Density of states (per unit area) : $D_2 = \dfrac{m}{2\pi\hbar^2}$

The degeneracy of Landau level (per unit area) is

$$N(B) = \hbar\omega_c D_2 = \frac{eB}{h} = \frac{1}{2\pi l^2} \ .$$

Cyclotron radius: $l = \sqrt{\dfrac{\hbar}{eB}}$

$\sim$ the radius of wavefunction

ϵ_F : Fermi energy in $B = 0$ ($\epsilon_F \sim 1$ eV)

n : Partially-occupied Landau level ($n \sim 10^3$ in $B = 1$T)

$$n\hbar\omega_c < \epsilon_F < (n+1)\hbar\omega_c$$

Landau level :

$$\epsilon_n(B) = \hbar\omega_c(B)\left(n + \frac{1}{2}\right)$$

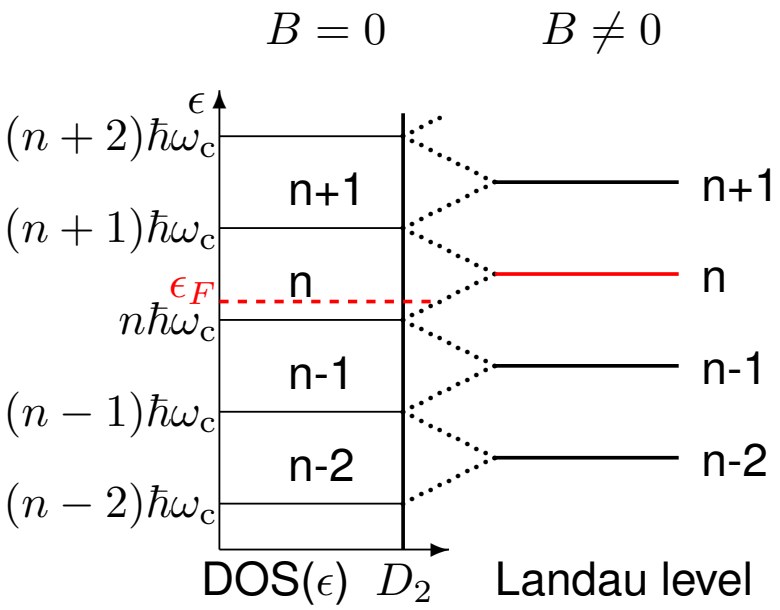
The increase of energy due to B : $E_2(B)$ ($E = E_0 + E_2(B)$)

$$E_2(B) = \int_{n\hbar\omega_c}^{\epsilon_F} \mathrm{d}\epsilon \left[\left(n + \frac{1}{2}\right)\hbar\omega_c - \epsilon\right] D_2$$

$$= \frac{1}{2}\left[(n+1)\hbar\omega_c - \epsilon_F\right]\left[\epsilon_F - n\hbar\omega_c\right] D_2$$

6.2 Landau levels in the k space

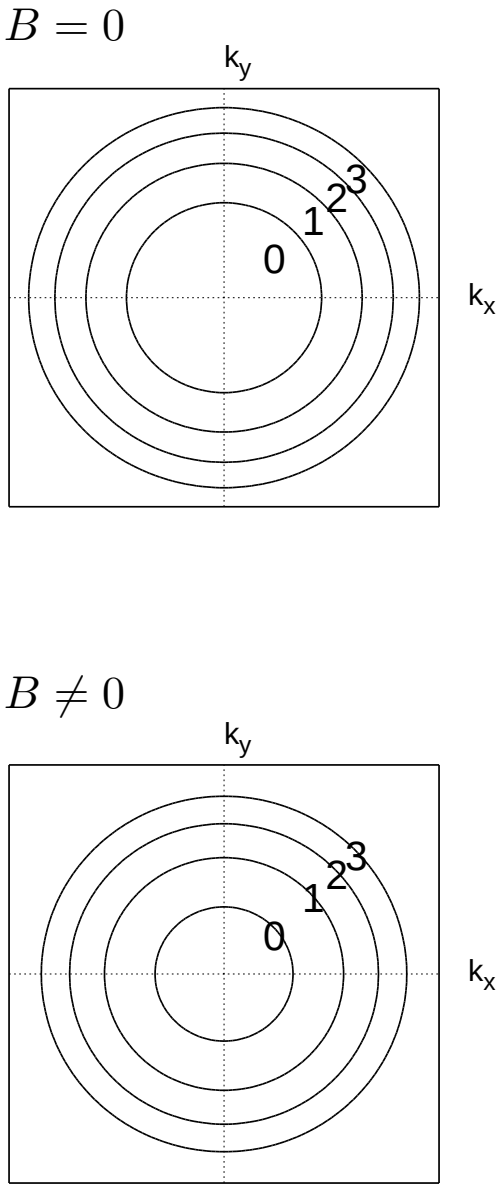
2-D free-electron system



Density of states: $D_2 = \dfrac{L_x L_y m}{2\pi\hbar^2}$

The degeneracy of Landau level :

$$N(B) = \hbar\omega_c D_2 = L_x L_y \frac{eB}{h}$$



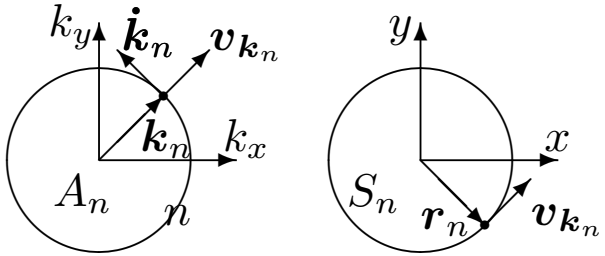
Landau loop (tube)

6.3 Semiclassical description of the n-th Landau-level electron motion

< A wave packet of the n-th Landau state >

$\boldsymbol{k}_n$: the k-space center of the wave packet

$\boldsymbol{r}_n$: the real-space center of the wave packet



$$\boldsymbol{v}_{\boldsymbol{k}_n} = \frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{k}_n}\frac{\hbar^2\boldsymbol{k}_n^2}{2m} = \frac{\hbar\boldsymbol{k}_n}{m}$$

$$\begin{aligned}\hbar\dot{\boldsymbol{k}}_n \equiv \hbar\frac{\mathrm{d}\boldsymbol{k}_n}{\mathrm{d}t} &= -e\boldsymbol{v}_{\boldsymbol{k}_n}\times\boldsymbol{B} \\ &= -e\frac{\hbar\boldsymbol{k}_n}{m}\times\boldsymbol{B} \\ |\dot{\boldsymbol{k}}_n| &= \frac{eB}{m}k_n = \omega_{\mathrm{c}}k_n\end{aligned}$$

$\boldsymbol{B}$: no work to the electron (W.P.)

W.P. moves on the equienergy surface.

Energy:

$$\frac{\hbar^2k_n^2}{2m} = \hbar\omega_{\mathrm{c}}\left(n+\frac{1}{2}\right)$$

Area of the Landau loop:

$$\begin{aligned}A_n = \pi k_n^2 &= \frac{2\pi eB}{\hbar}\left(n+\frac{1}{2}\right) \\ &= \frac{4\pi^2B}{\phi_0}\left(n+\frac{1}{2}\right) \quad ; \quad \phi_0 = \frac{h}{e}\end{aligned}$$

In real space,

$$\begin{aligned}\omega_{\mathrm{c}}r_n = |\boldsymbol{v}_{\boldsymbol{k}_n}| &= \frac{\hbar k_n}{m} \\ r_n = \frac{\hbar}{m\omega_{\mathrm{c}}}k_n &= \frac{\phi_0}{2\pi B}k_n \\ S_n = \pi r_n^2 = \pi\left(\frac{\hbar k_n}{\omega_{\mathrm{c}}m}\right)^2 &= \frac{\phi_0^2}{4\pi^2B^2}A_n \\ &= \frac{\phi_0}{B}\left(n+\frac{1}{2}\right)\end{aligned}$$

7 Landau diamagnetic susceptibility (Landau 反磁性)

($T \sim 300$K, $k_{\rm B}T \sim 30$meV, $B \sim 10^{-2}$T, $\hbar\omega_{\rm c} \sim 10^{-2}$meV, $\epsilon_{\rm F} \sim 1$eV, $n \sim 10^5$)

$k_{\rm B}T \gg \hbar\omega_{\rm c}$

average over various Fermi energy : $n\hbar\omega_{\rm c} \le \epsilon_{\rm F} \le (n+1)\hbar\omega_{\rm c}$

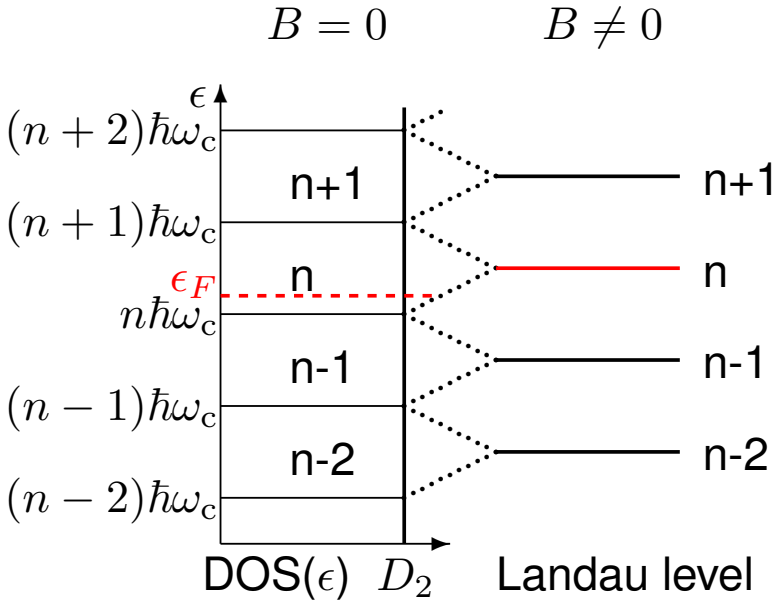
$$\langle E_2(B)\rangle = \frac{1}{\hbar\omega_{\rm c}} \int_{n\hbar\omega_{\rm c}}^{(n+1)\hbar\omega_{\rm c}} {\rm d}\epsilon_{\rm F}\, E_2(B) = \frac{1}{3}\mu_{\rm B}^2 D(\epsilon_{\rm F}) B^2$$

$$M(B) = -\frac{{\rm d}\langle E_2(B)\rangle}{{\rm d}B}$$

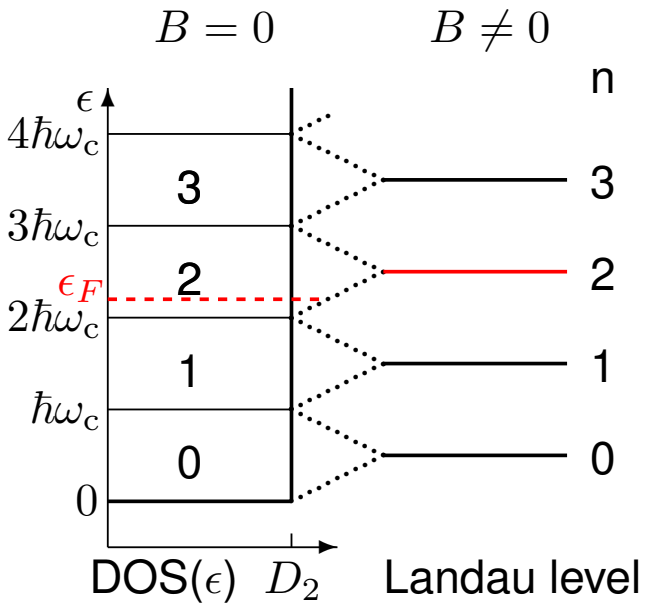
$$\chi_{\rm L} = \frac{\mu_0 M(B)}{B} = -\frac{2}{3}\mu_0\mu_{\rm B}^2 D(\epsilon_{\rm F}) = -\frac{1}{3}\chi_{\rm P}$$

$\chi_{\rm L}$ Landau susceptibility

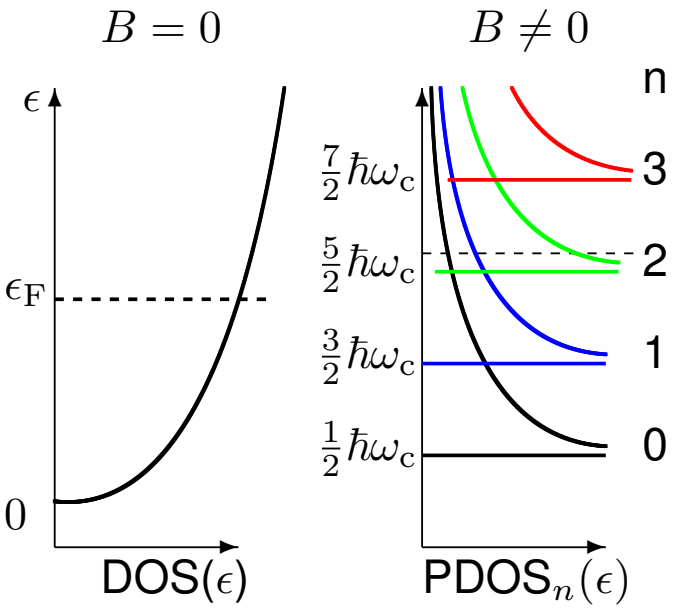
$\chi_{\rm P}$ Pauli susceptibility



2-D free-electron system

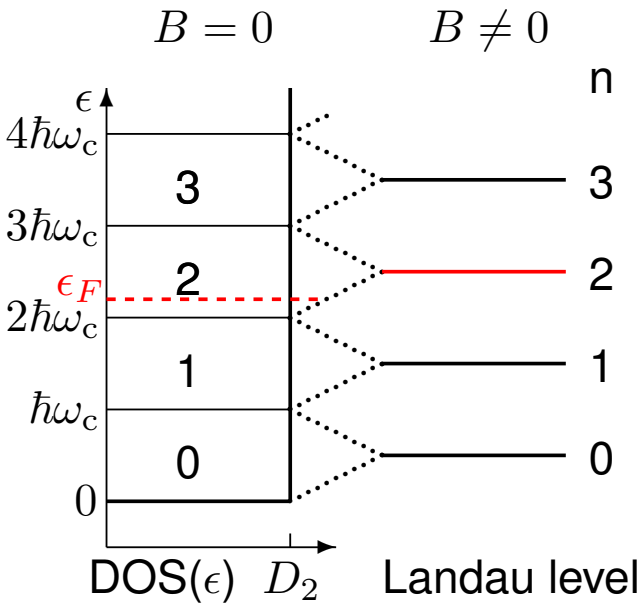


2-D free-electron system



3-D free-electron system

8 de Haas - van Alphen effect



2-D free-electron system

2-D free-electron system

n : Partially occupied Landau level

$$n\hbar\omega_c < \epsilon_F < (n+1)\hbar\omega_c \quad ; \quad \omega_c = \frac{eB}{m}$$

$$n < \frac{B_1}{B} < n+1 \quad ; \quad \frac{\epsilon_F}{\hbar\omega_c} = \frac{m\epsilon_F}{\hbar eB} = \frac{B_1}{B}, \quad B_1 = \frac{m\epsilon_F}{\hbar e}$$

$$\frac{n}{B_1} < \frac{1}{B} < \frac{n+1}{B_1}$$

The increase of energy due to B :

$$E_2(B) = \int_{n\hbar\omega_c}^{\epsilon_F} \mathrm{d}\epsilon \left[\left(n+\frac{1}{2}\right)\hbar\omega_c - \epsilon\right] D_2 = \frac{1}{2}\left[(n+1)\hbar\omega_c - \epsilon_F\right]\left[\epsilon_F - n\hbar\omega_c\right] D_2$$

Magnetization : $\left(\mu_B = \frac{e\hbar}{2m}\right), \quad M_0 = \mu_B \epsilon_F D_2$

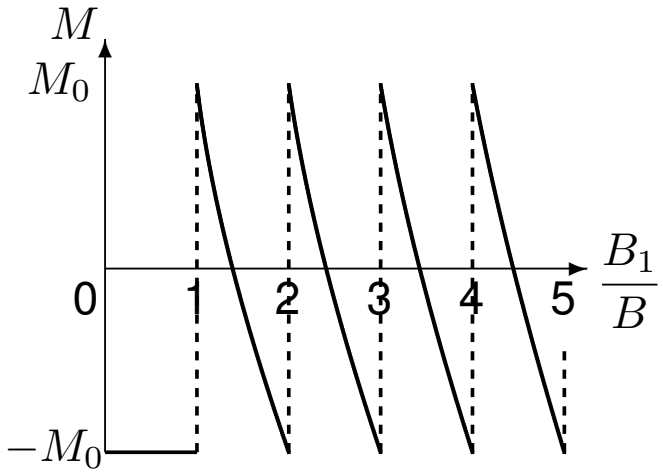
$$M(B) = -\frac{\partial E_2(B)}{\partial B} = -\mu_B\left[(\epsilon_F - n\hbar\omega_c)(n+1) - ((n+1)\hbar\omega_c - \epsilon_F)n\right] D_2$$

$$= -\mu_B \epsilon_F D_2 B\left[\left(\frac{1}{B} - \frac{n}{B_1}\right)(n+1) - \left(\frac{n+1}{B_1} - \frac{1}{B}\right)n\right]$$

de Haas - van Alphen effect

Magnetization oscillates as a function of B.

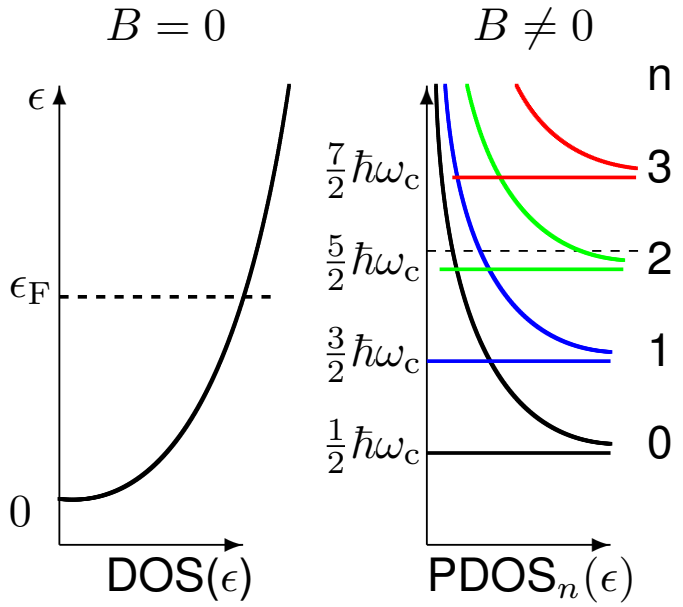
In the 2-D free-electron system,



$M_0 = \mu_{\rm B}\epsilon_{\rm F}D_2$

$\hbar\omega_{\rm c} = \epsilon_{\rm F}$ when $B = B_1$

In the 3-dimensional system,



DOS($B \neq 0$) : still oscillatory

$\Downarrow$

oscillation of $1/B$ plot

$\Rightarrow$ Fermi surface

(extremal orbit)

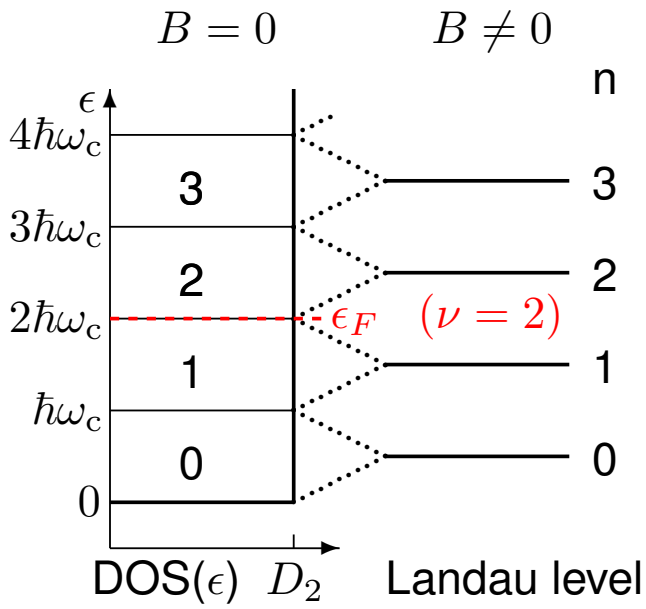
$T \sim 1$K, $k_{\rm B}T \sim 10^{-1}$meV, $B \sim 10$T, $\hbar\omega_{\rm c} \sim 10$meV, $\epsilon_{\rm F} \sim 1$eV, $n \sim 100$

9 Quantum Hall effect (2D)

For the electric field $E_x \neq 0$ and $E_y = 0$, the center of the cyclotron motion moves in the y direction with the velocity $v_{\rm c} = \dfrac{E_x}{B}$, which causes the Hall current

$$J_y = ev_c N_{\rm e} = \frac{eN_{\rm e}}{B} E_x \ ,$$

where $N_{\rm e}$ is the total electron number per unit area. Namely, the Hall conductivity is $\boxed{\sigma_{yx} = \dfrac{eN_{\rm e}}{B}}$



2-D free-electron system

When $\dfrac{N_{\rm e}}{N(B)} = \nu \quad (\nu = 1, 2, \cdots)$, the system becomes insulator. The condition gives $\dfrac{N_{\rm e} h}{eB} = \nu$; namely, $\boxed{\dfrac{eN_{\rm e}}{B} = \nu \dfrac{e^2}{h}}$.

Thus, for the insulating state, the Hall conductivity is quantized:

$$\sigma_{yx} = \nu \frac{e^2}{h} \quad (\nu = 1, 2, 3, \cdots)$$

ν: TKNN number (1982), which is proved to be topological invariant.