

Quantum simulation of atom motion in materials

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^B Institute of Industrial Science (IIS), the University of Tokyo

^C Graduate School of Engineering, Osaka University

^D Osaka University

What is Naniwa ?

- “Naniwa” is the former name of Osaka, Japan



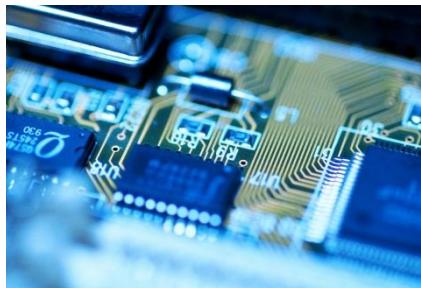
- NANIWA is the name of our simulation code.

浪速

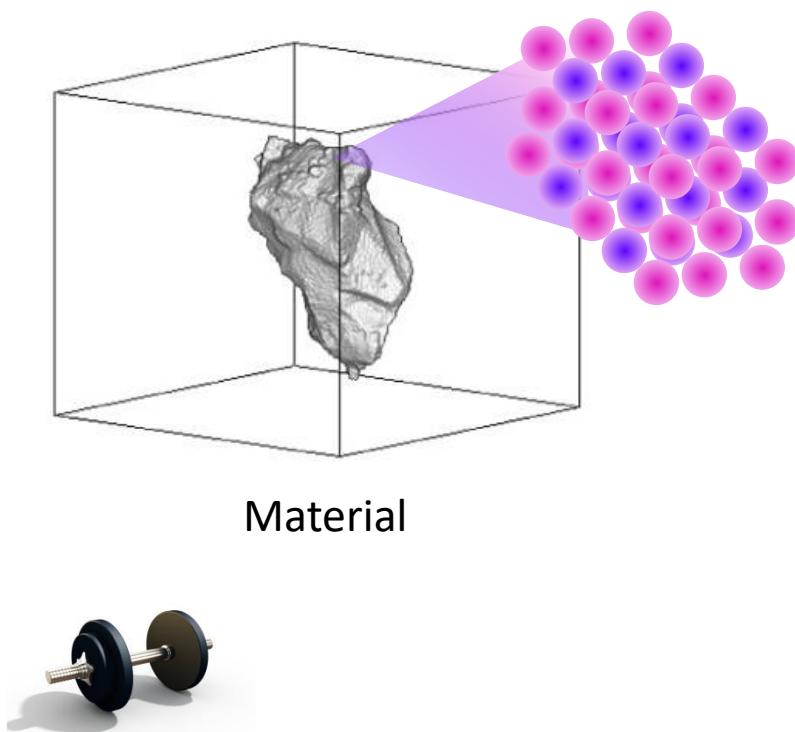
Naniwa

- The fast wave

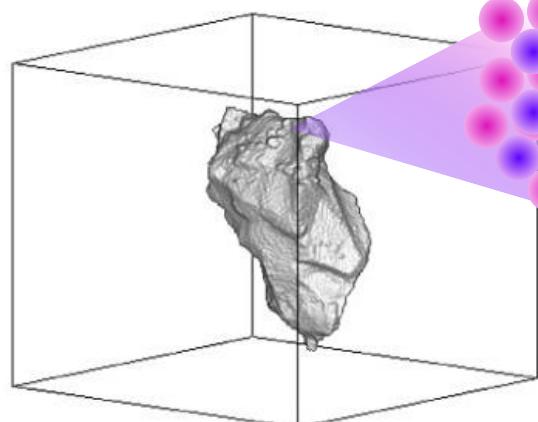
materials



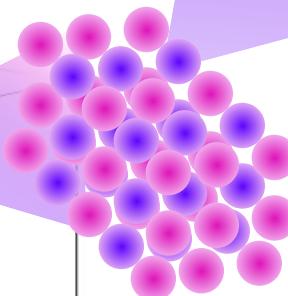
Particles which construct the materials



Particles which construct the materials



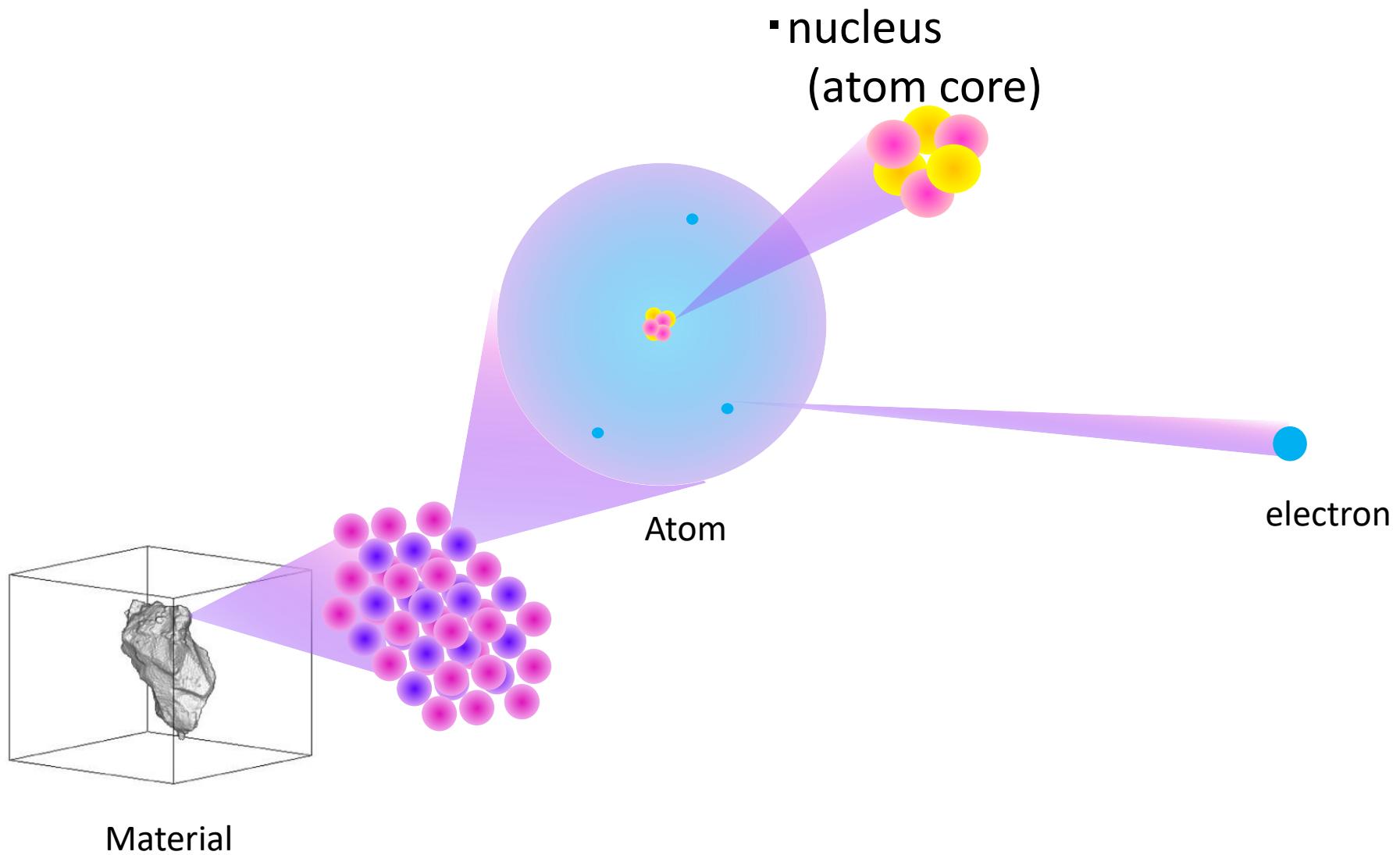
Material



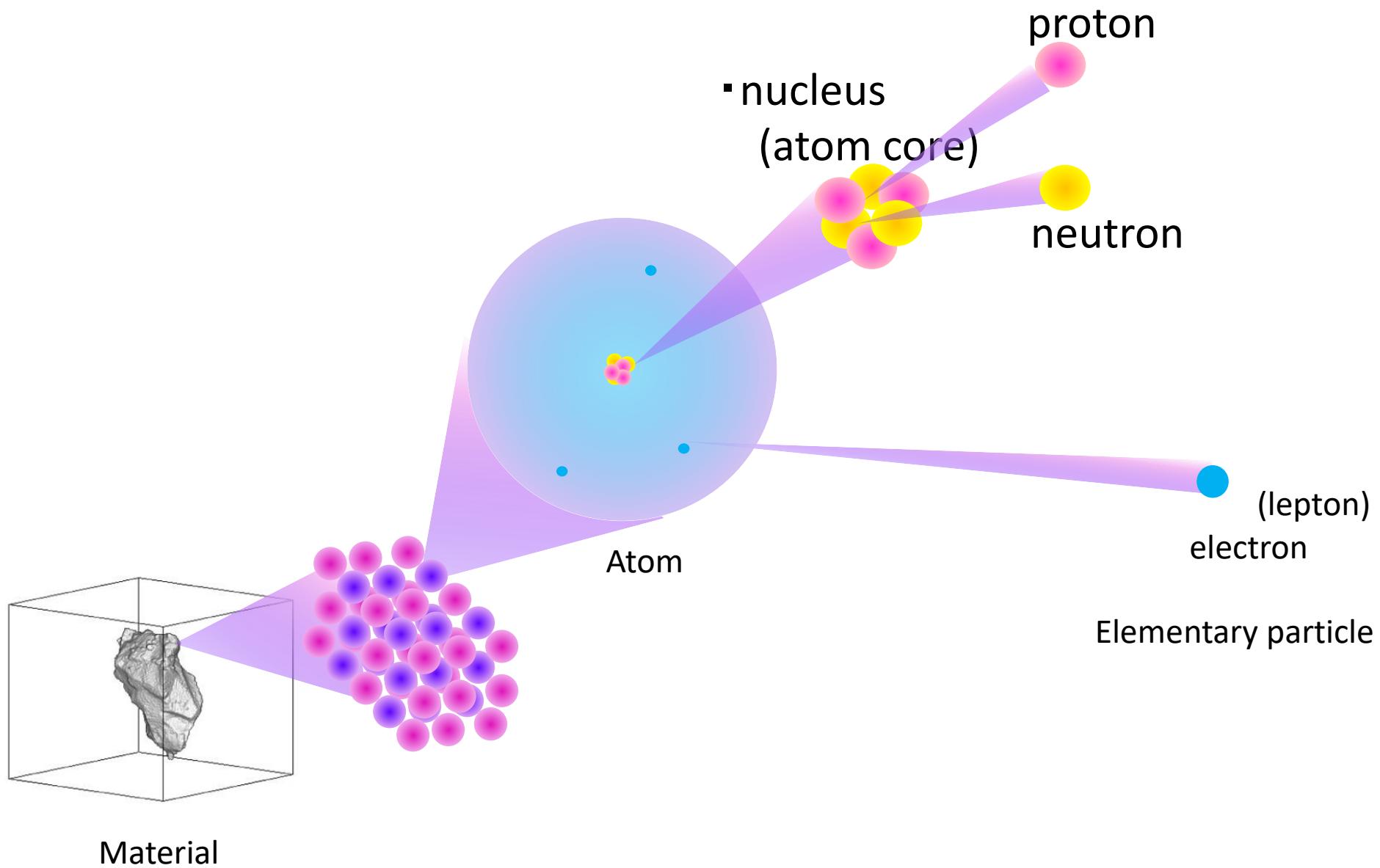
Atom

$$1 \text{ \AA} = 0.1 \text{ nm} \\ = 10^{-10} \text{ m}$$

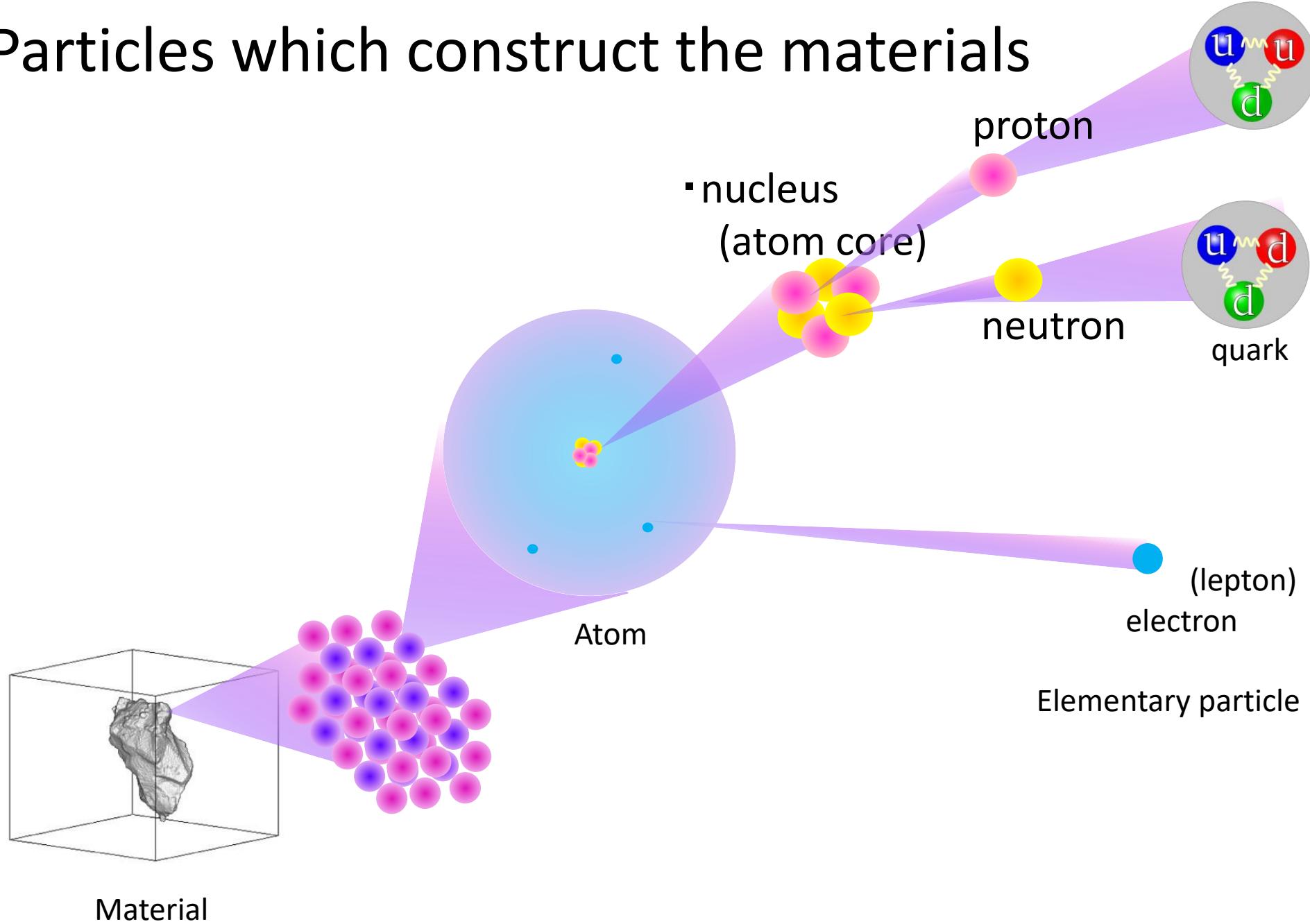
Particles which construct the materials



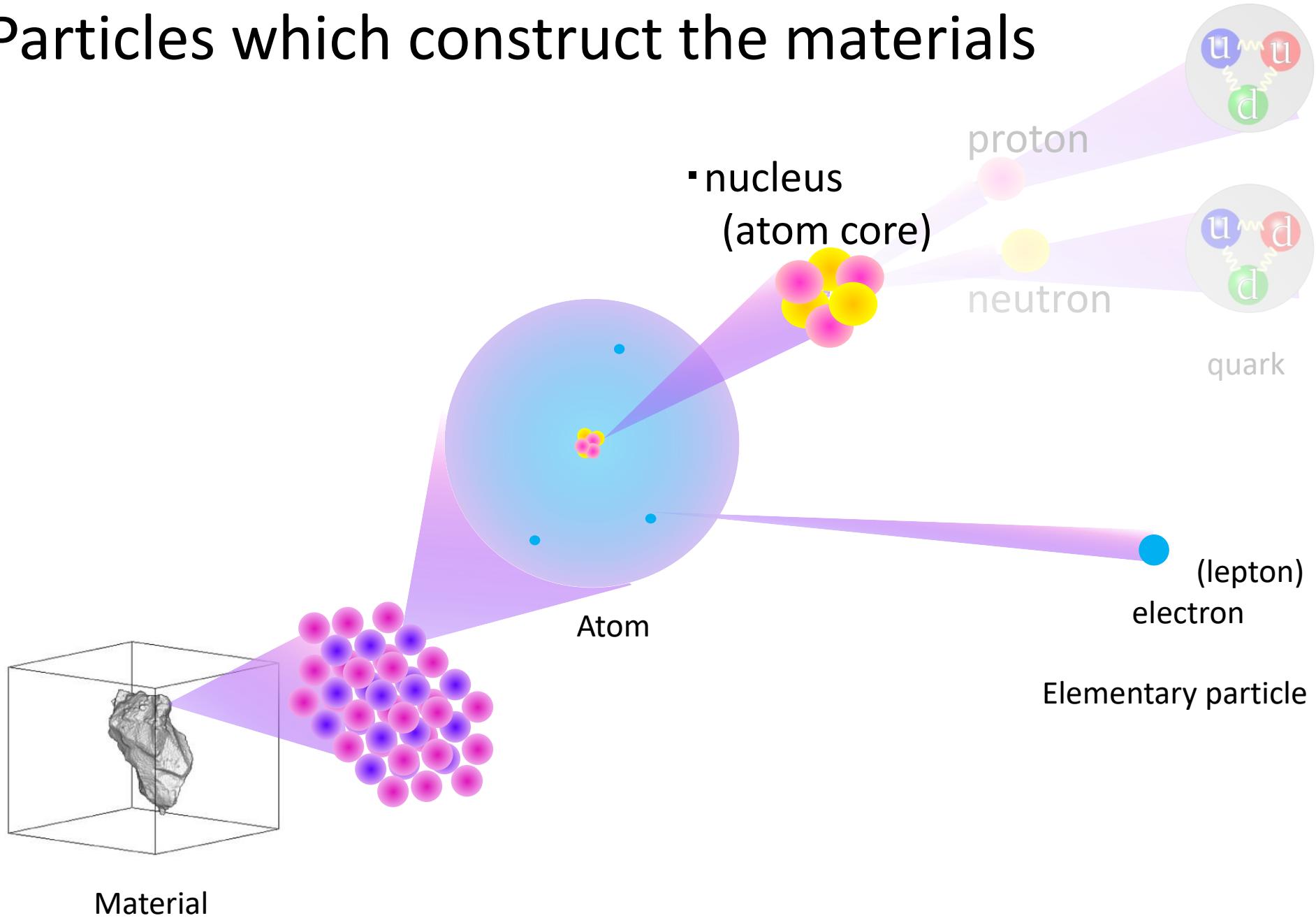
Particles which construct the materials



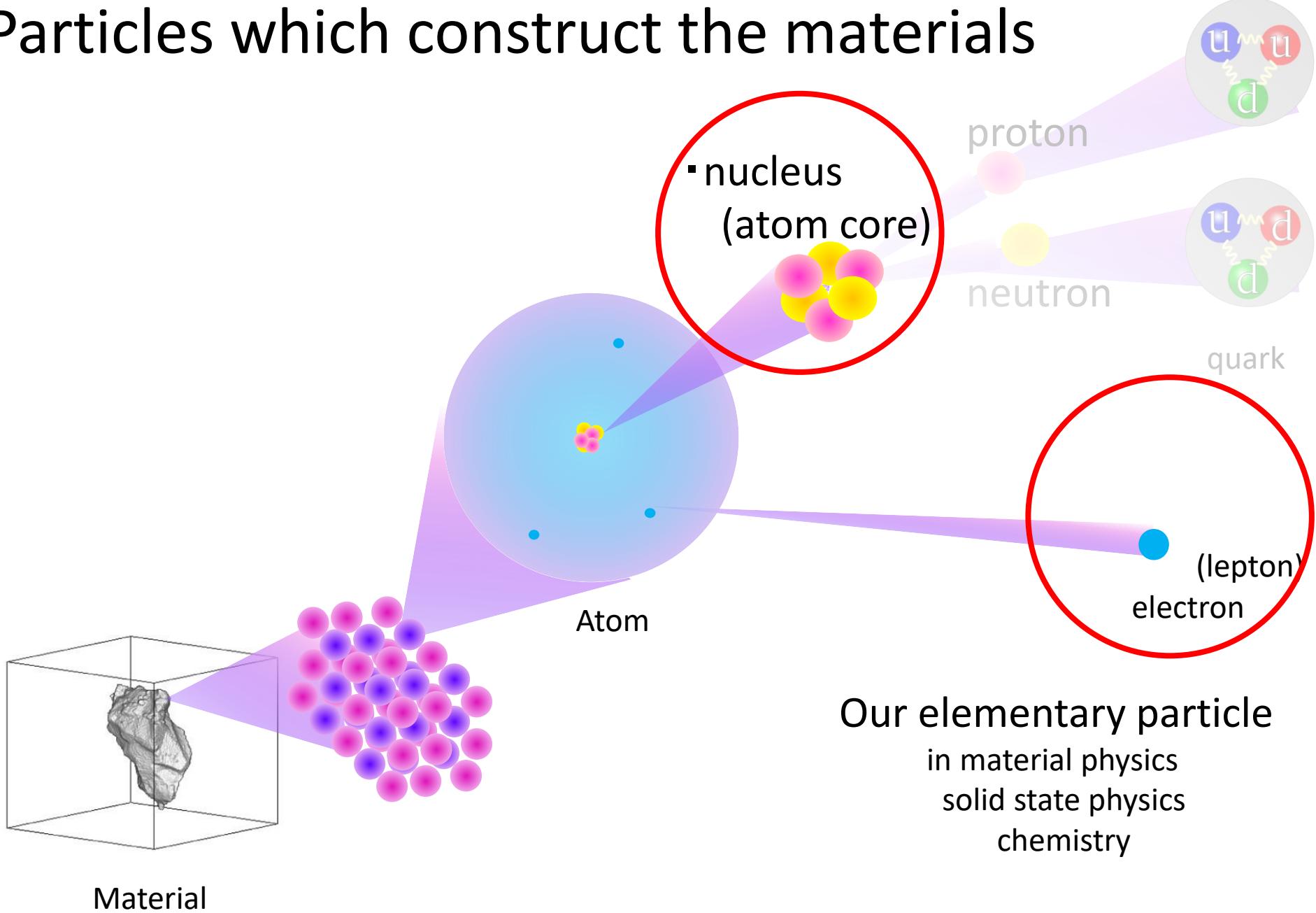
Particles which construct the materials



Particles which construct the materials



Particles which construct the materials

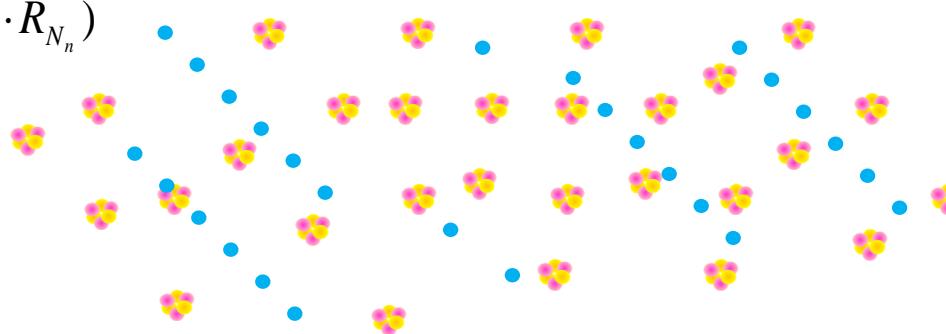


Many atomic nuclei and electrons are in the material

- Fundamental equation for describing behaviors of these particles:
Total Schrödinger equation: *partial differential equation*

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

$$= E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$



N_n ; Number of atoms

N_e ; Number of electrons $\sim 10^{23}$

Z_l : atomic number of the l th atom

● electron

M_l : Mass of the l th atom

● atomic nucleus

m : Electron mass

Many atomic nuclei and electrons are in the material

- Fundamental equation for describing behaviors of these particles: Total Schrödinger equation

Hamiltoinan:

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right]$$

Wave function

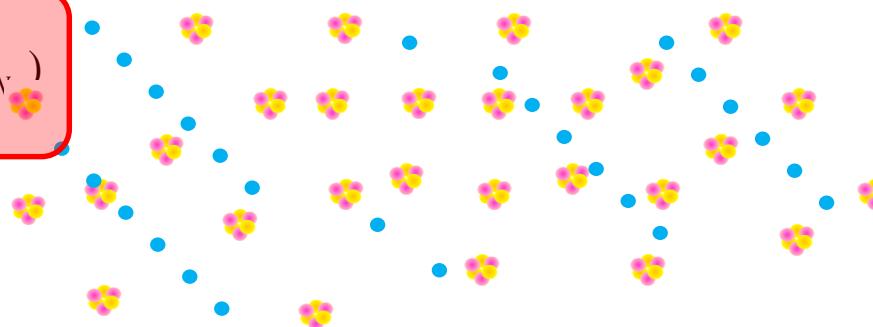
$$\psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

= E

$$\psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

As a function of all particle positions

Total energy



N_n ; Number of atoms

N_e ; Number of electrons $\sim 10^{23}$

Z_l : atomic number of the l th atom

electron

M_l : Mass of the l th atom

atomic nucleus

m : Electron mass

Hamiltoinan = Energy operator for all particles

● electron



atomic nucleus

①

②

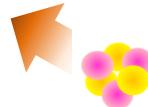
③

④

⑤

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) \\ = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

①Kinetic energy
of nucleus



②Kinetic energy
of electron



③electron–nucleus
interaction potential energy



④ electron–electron
interaction potential energy



⑤ nucleus–nucleus
interaction potential energy



①

②

③

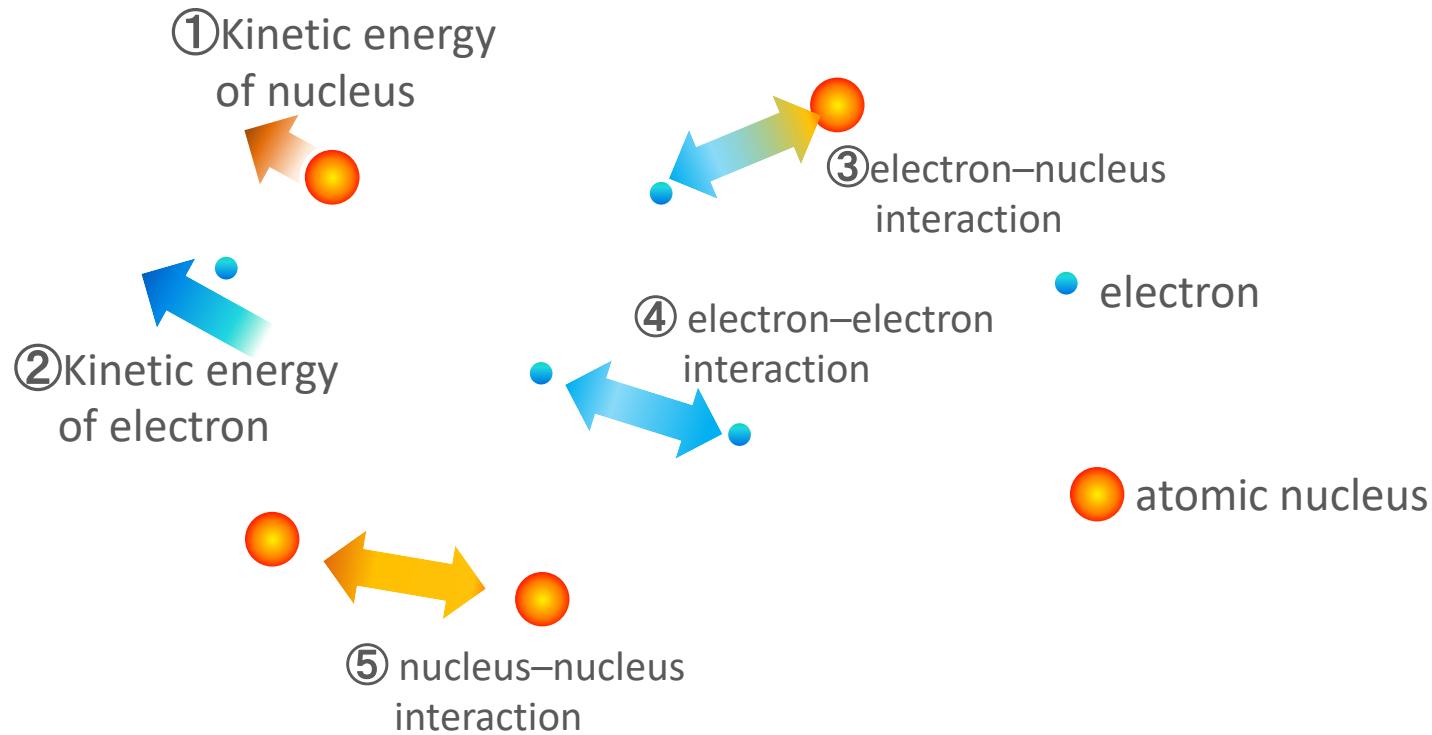
④

⑤

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

$$= E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

If $M_l \gg m$ then ...



①

②

③

④

⑤

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

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If $M_l \gg m$ then ...

① Kinetic energy of nucleus



② Kinetic energy of electron



From the view point of electron motion,
we can treat that nuclei does not move.



③ electron–nucleus
interaction potential energy



④ electron–electron
interaction potential energy



⑤ nucleus–nucleus
interaction potential energy

①

②

③

④

⑤

$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

If $M_l \gg m$ then ...

① Kinetic energy of nucleus



② Kinetic energy of electron



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③ electron–nucleus
interaction potential energy



④ electron–electron
interaction potential energy



⑤ nucleus–nucleus
interaction potential energy

- Fundamental equation for describing behaviors of electrons:
Schrödinger equation

$$\left[-\frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

- Information about the electron state in the material systems
→ Material properties

Z_l : atomic number of the l^{th} atom

N_n ; Number of atoms

N_e ; Number of electrons ~ 10^{24}

Density functional theory based ab initio calculation

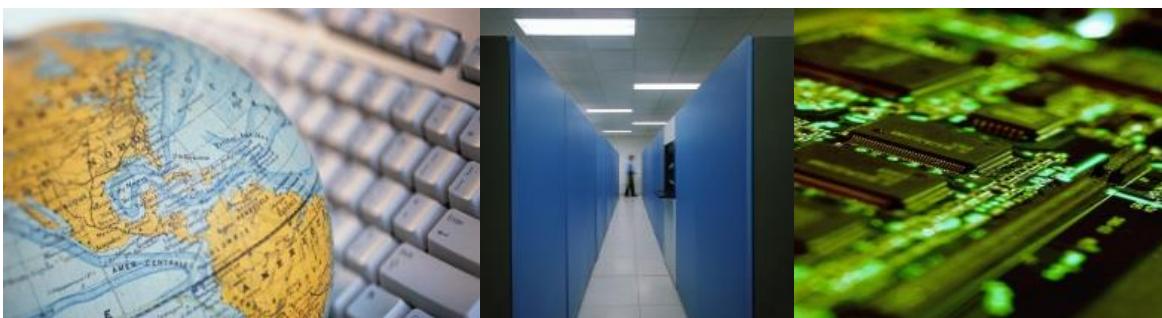
Density functional Theory (DFT)

- Walter Kohn, Pierre Hohenberg (1964)

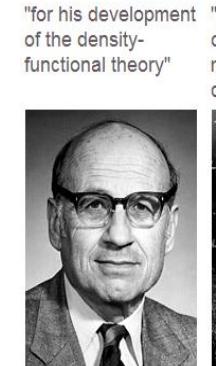
- Hohenberg-Kohn theorems (H-K).
- Kohn-Sham DFT
- ▪ ▪ ▪
- local-density approximation (LDA)
- Generalized gradient approximations (GGA)

+

rapid progress of technological innovation
in Computer Science: Computics



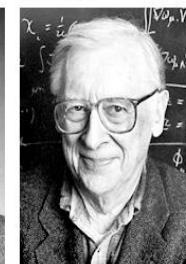
The Nobel Prize in Chemistry 1998



Walter Kohn

1/2 of the prize

USA



John A. Pople

1/2 of the prize

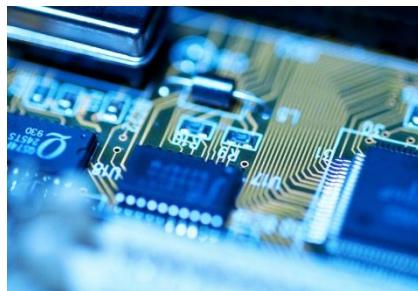
United Kingdom

Walter Kohn & John A. Pople
<http://nobelprize.org/>

Unique equation

$$\left[-\frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

Various properties



$$\left[-\frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1 (i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1 (l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$

Aluminum Al(Z=13)

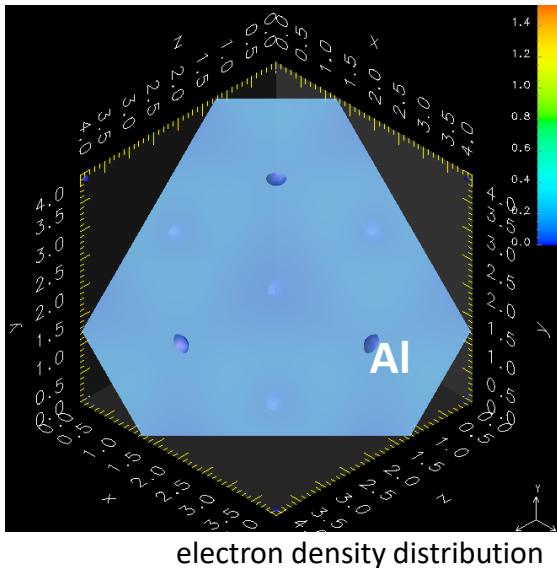
Silicon Si (Z=14)

Sodium chloride Na (z=11) Cl (z=17)

Periodic table

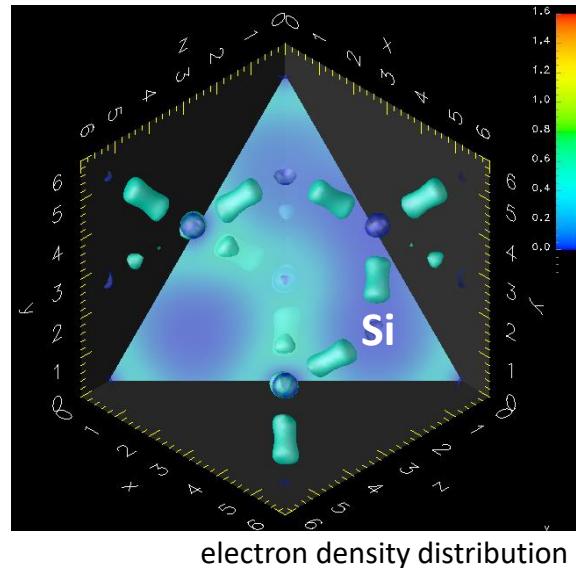
Aluminum

Al ($Z=13$)



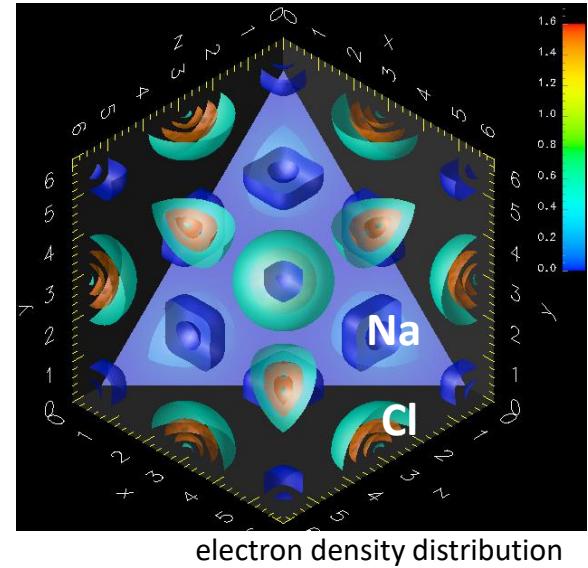
Silicon

Si ($Z=14$)



Sodium chloride

Na ($Z=11$) Cl ($Z=17$)



Metallic bond

delocalized electrons
in the crystal

Covalent bond

Localized electrons
between atoms

Ionic bond

Localized electrons
at atoms

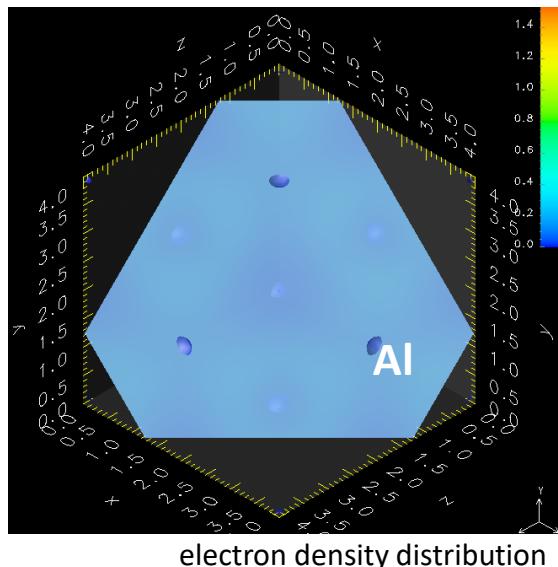
Na⁺ ions, Cl⁻ ions

DFT-based ab initio calculation method is one of the most successful method.

Parameter in the calculations is only atomic number, Z.

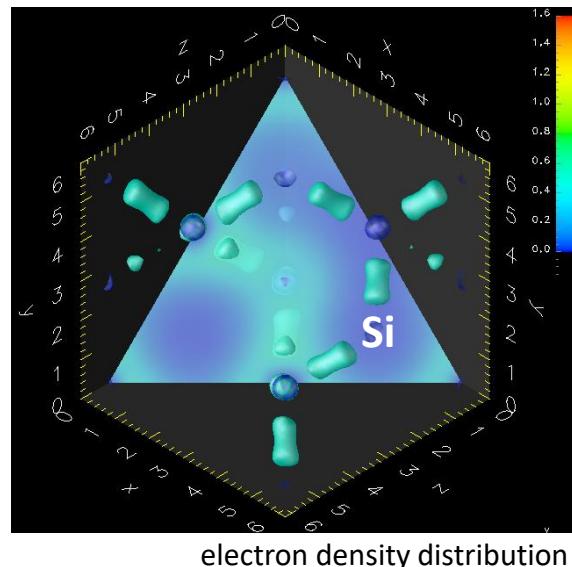
Aluminum

Al ($Z=13$)



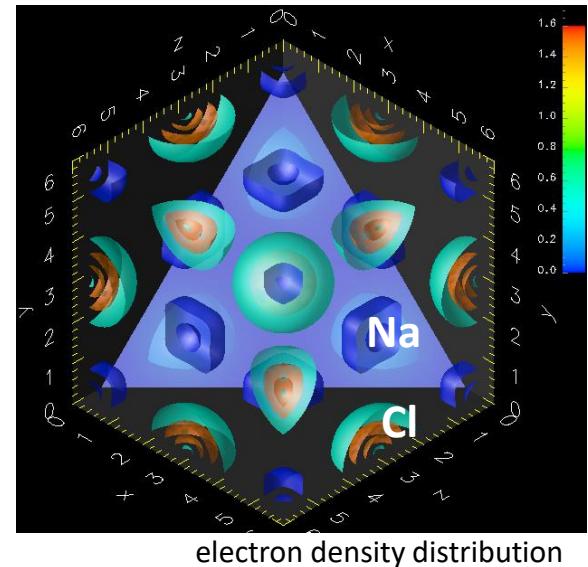
Silicon

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Metallic bond

delocalized electrons
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Covalent bond

Localized electrons
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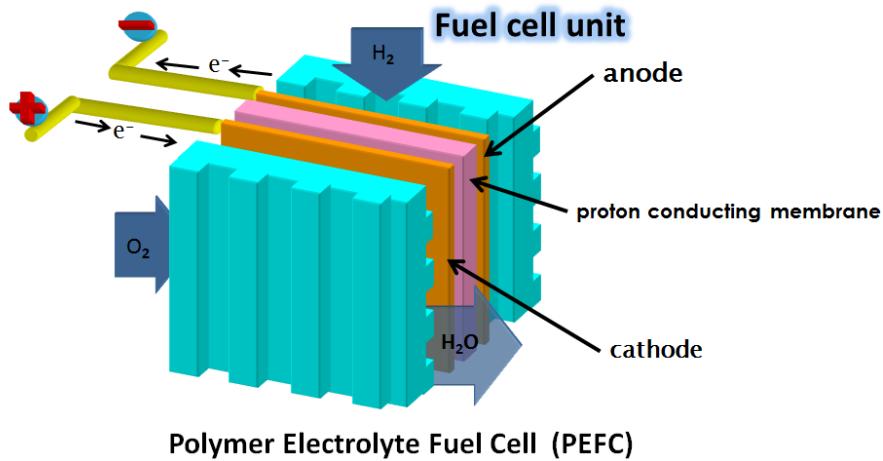
Ionic bond

Localized electrons
at atoms

Na⁺ ions, Cl⁻ ions

We want to treat the behavior of the hydrogen in the material.
Hydrogen is one of the key element for the future technology.
“energy technology”, “biotechnology”

Hydrogen Reactions on Fuel cell anode

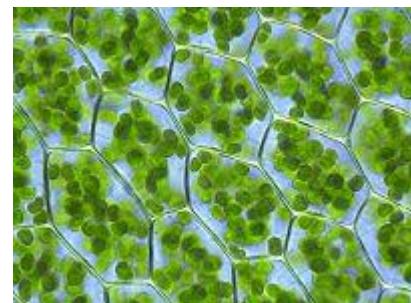
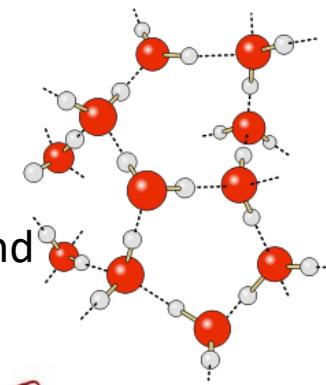


$$\cancel{M_{\text{Proton}}} \gg m$$

electrolyte

hydrogen bond

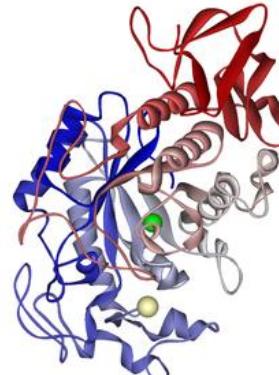
aqueous solution



Photosynthesis

biochemistry

Enzyme catalysis



①

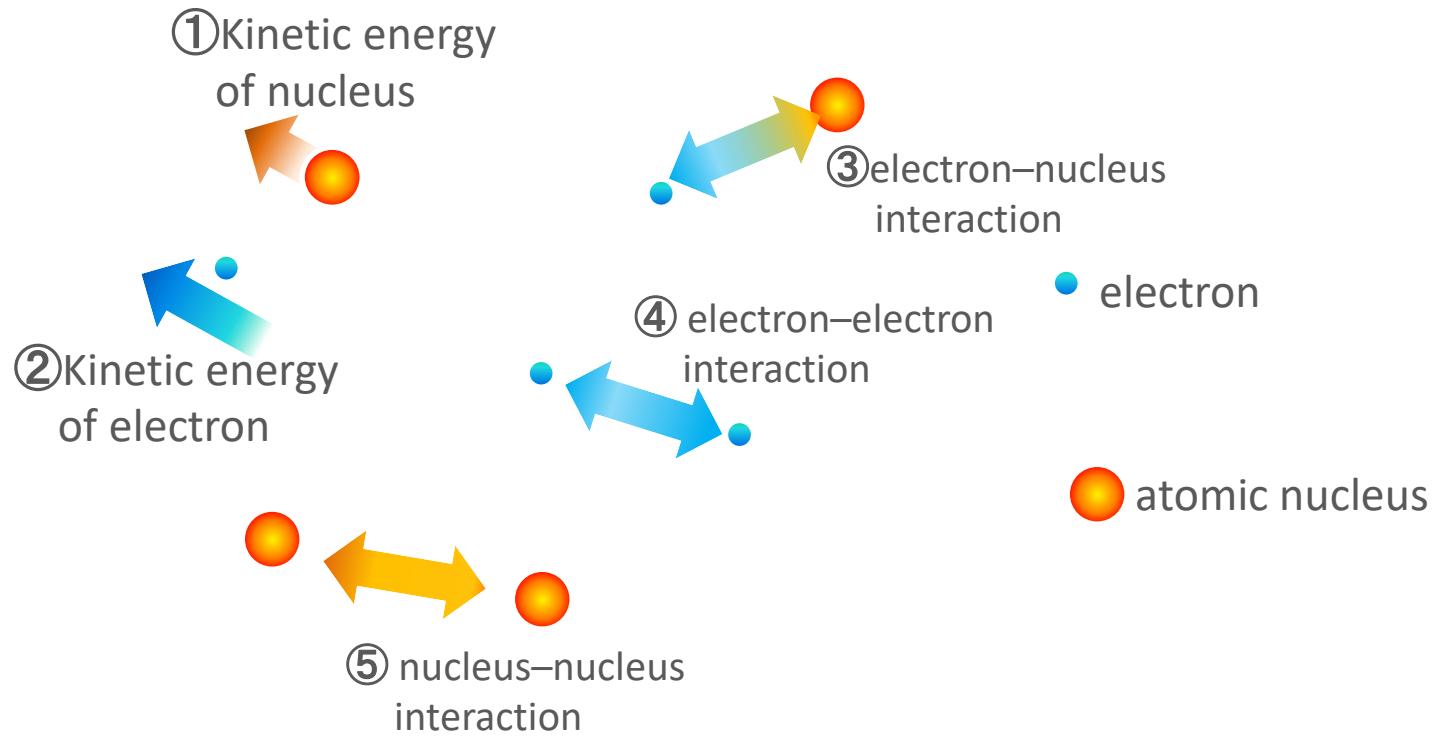
②

③

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$$\left[-\sum_{l=1}^{N_n} \frac{\hbar^2}{2M_l} \nabla_i^2 - \frac{\hbar^2}{2m} \sum_{i=1}^{N_e} \nabla_i^2 - \sum_{i=1}^{N_e} \sum_{l=1}^{N_n} \frac{Z_l e^2}{|\vec{r}_i - \vec{R}_l|} + \frac{1}{2} \sum_{i=1}^{N_e} \sum_{j=1(i \neq j)}^{N_e} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sum_{l=1}^{N_n} \sum_{k=1(l \neq k)}^{N_n} \frac{Z_l Z_k e^2}{|\vec{R}_l - \vec{R}_k|} \right] \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n}) \\ = E \psi(\vec{r}_1, \dots, \vec{r}_{N_e}; \vec{R}_1, \dots, \vec{R}_{N_n})$$



What is Naniwa ?

- **NANIWA** is a computational code for performing first principles quantum mechanical calculations.
- Two kinds of Naniwa codes in Kasai lab.

Naniwa for quantum reaction: It is a quantum mechanical version of the first principles molecular dynamics (MD) calculations, for reactions.

“we can solve the scattering problems, and obtain the probability of some events, adsorption, desorption, reflection, excitation, etc.”

Naniwa for quantum state: It is a nucleus version of the first principles quantum state calculations.

“We can solve the eigenvalue problem, and obtain the eigenstates and their eigenenergies for atom (nuclear) motion”

Naniwa codes

- We have been developing the quantum simulation code “Naniwa” for the small mass atoms, H, Li, ..., on the solid surface, in the subsurface as well as in the bulk.


$$^1_1\text{H}$$

Protium


$$^2_1\text{H}$$

Deuterium


$$^3_1\text{H}$$

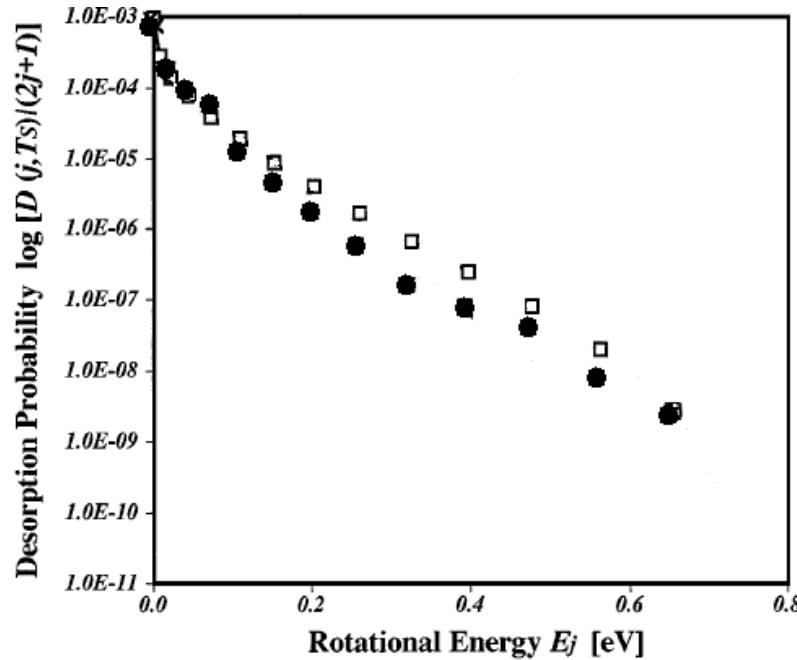
Tritium


$$^7_2\text{Li}$$

Lithium

...

Rotational energy distributions of hydrogen molecule desorbing from Cu(111) surface



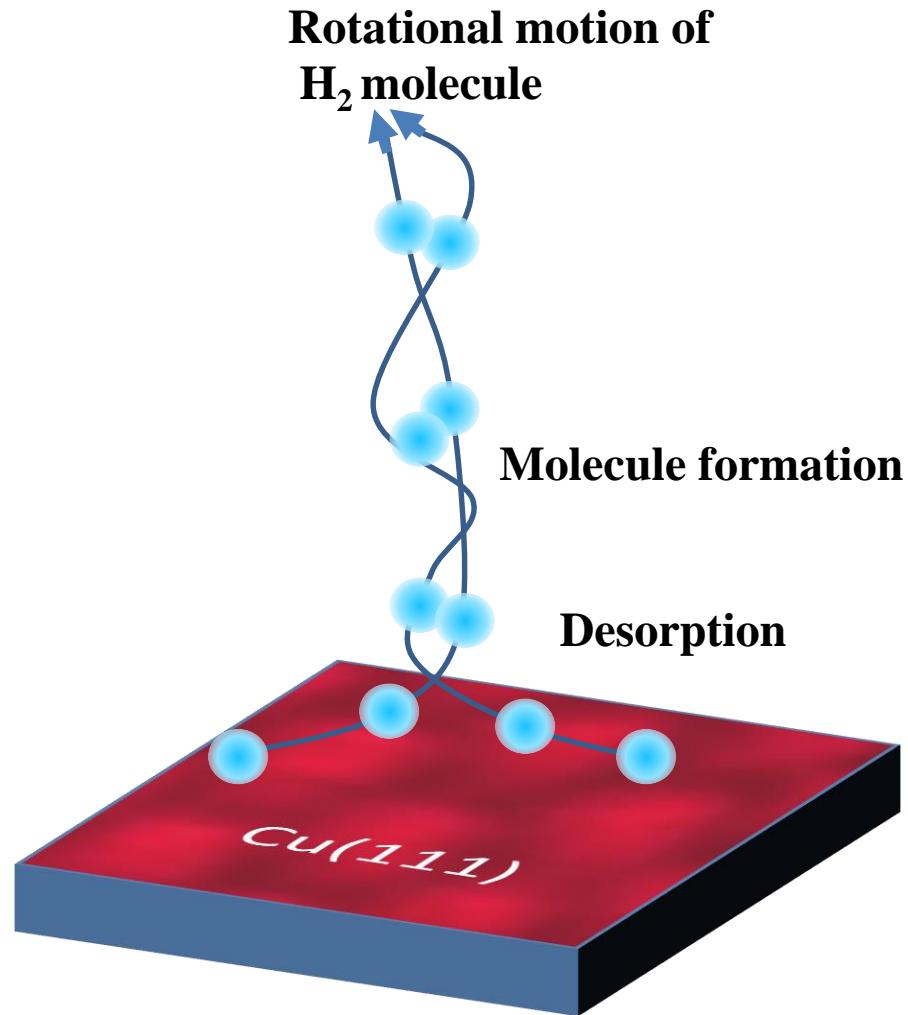
Our simulation results \square square

: W.A.Dino, *et al.*, PRL **78** (1997) 286.

Experimental data \bullet circle

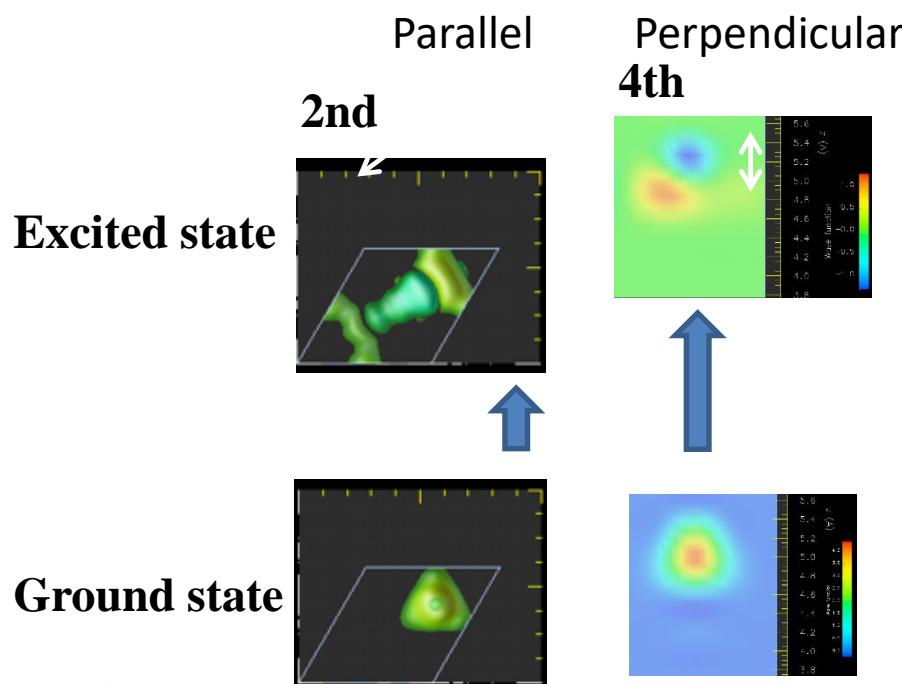
: H.A.Michelsen *et al.*, J. Chem. Phys. **98**(1993) 8294 .

No fitting parameters !



Surface vibration energy of Hydrogen atom adsorbed on Pd(111) surface

Motion	Parallel	Perpendicular
Experiment*	96 meV	124 meV
Simulation	91 meV	114 meV
Error	5.2%	8.1%



No fitting parameters !

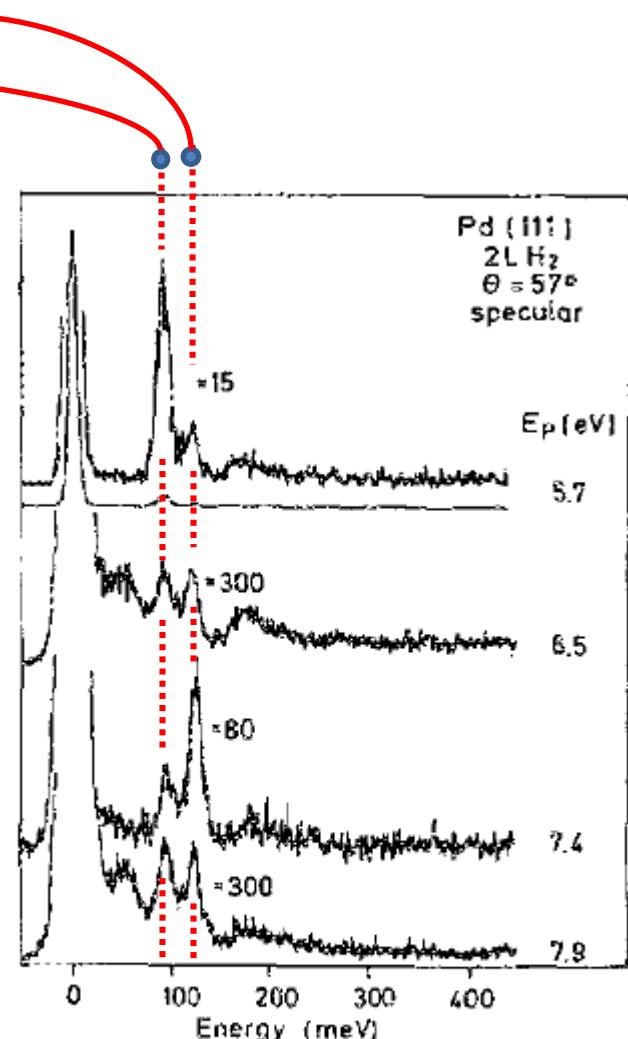


FIG. 1. HREELS of H on Pd(111)

Electron energy loss spectroscopy (EELS)*

H. Conrad, et al, J. Vac. Sci. Technol. A5, 452 (1987).

Comparison between our simulation Naniwa and experimental results

Surface-normal vibration excitation energy of hydrogen atom adsorbed on metal surface

	Naniwa	Experiment	Error
H on Pd(111)	114 meV	124 meV**	8.1%

** H. Conrad, *et al*, J. Vac. Sci. Technol. A5, 452 (1987).

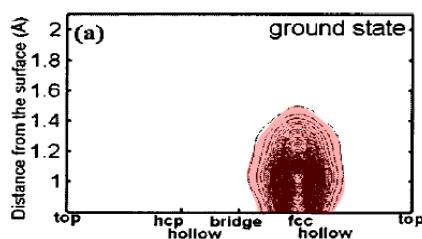
	Naniwa	Experiment	Error
H on Cu(111)	135 meV	129 meV*	4.7%
D on Cu(111)	104 meV	96 meV	8.3%
isotope effect	1.29	1.34	3.4%

*G. Lee, *et al.*, Surf. Sci. 498 (2002) 229.

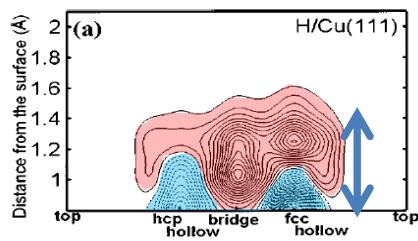
Less than 10% error

H on Cu(111)

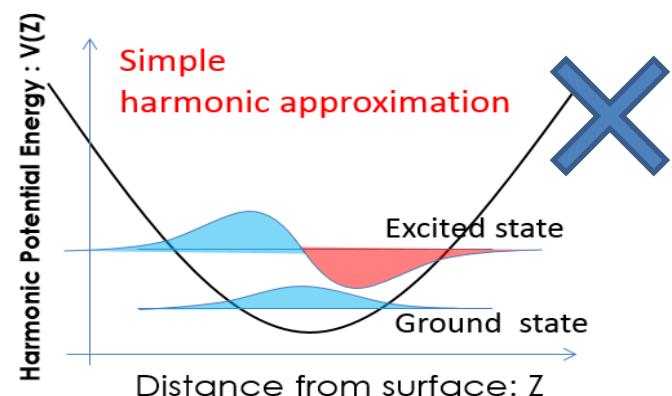
Ground state



Vibrational excited state



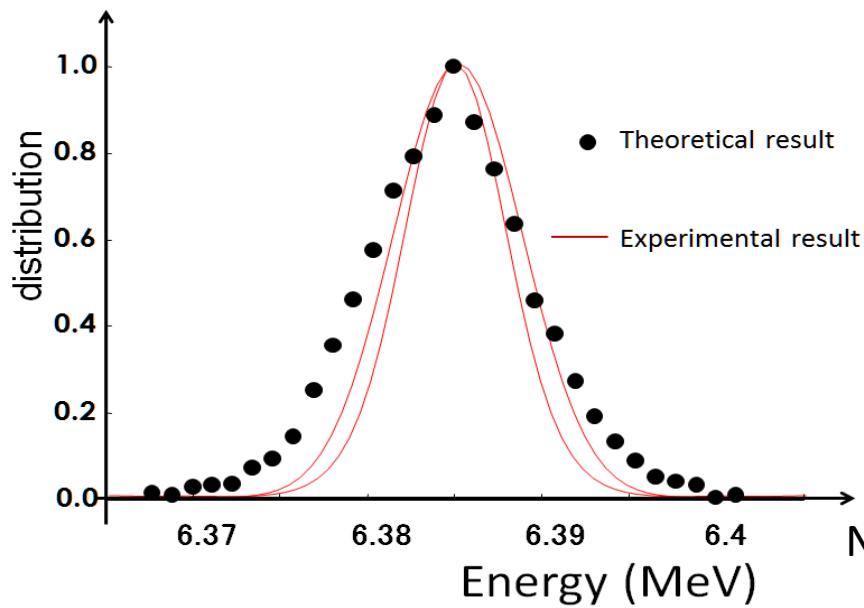
No fitting parameters !



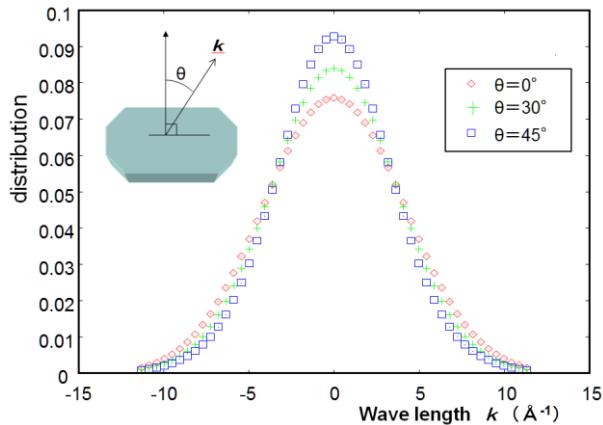
$$\text{isotope effect ratio} = \sqrt{2} \sim 1.414$$

Comparison between our simulation Naniwa and experimental results

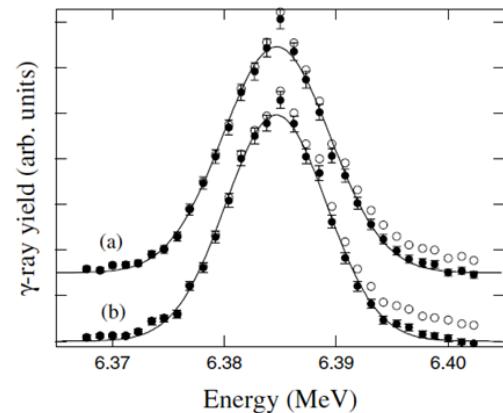
Momentum distribution of adsorbed hydrogen atom is observable in its ground state.



NRA : Nuclear Reaction Analysis



Naniwa

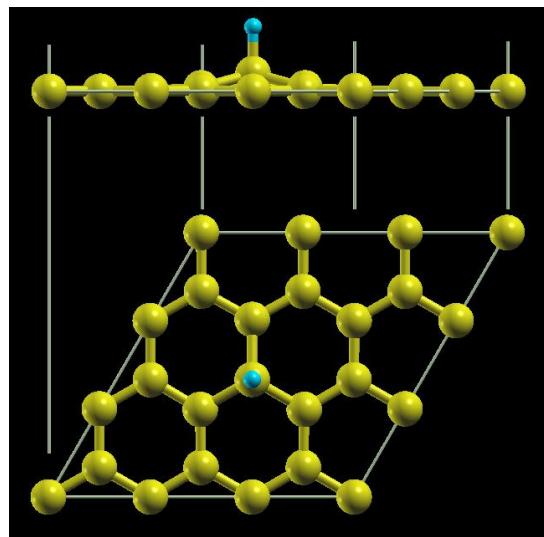


K. Fukutani et al., PRL 48(2002)116101.

Comparison between our simulation Naniwa and experimental results

Hydrogen adsorbed on graphene

Conventional ab initio cal.

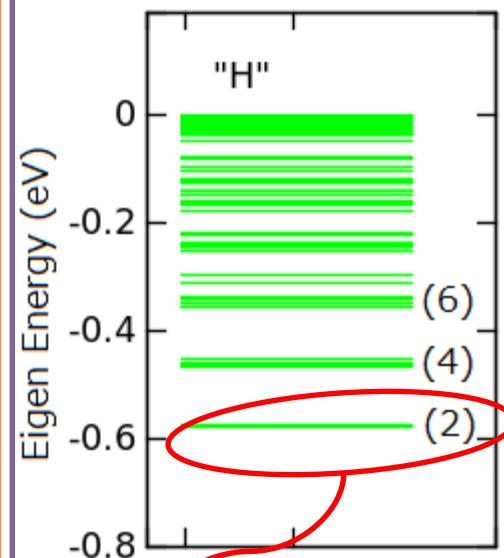


Adsorbed site: Top site

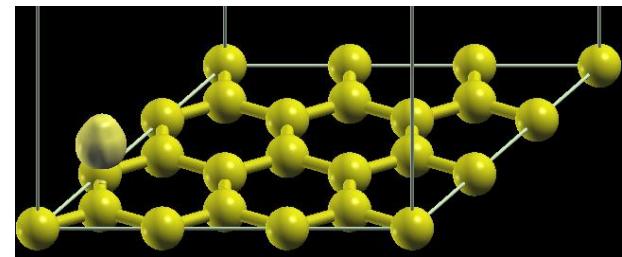
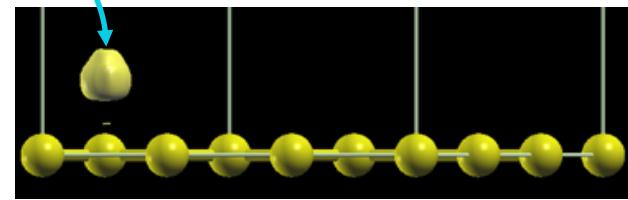
Adsorbed type: chemisorption

Adsorbed energy: **-0.816 eV**

Naniwa results



Ground state Wave function

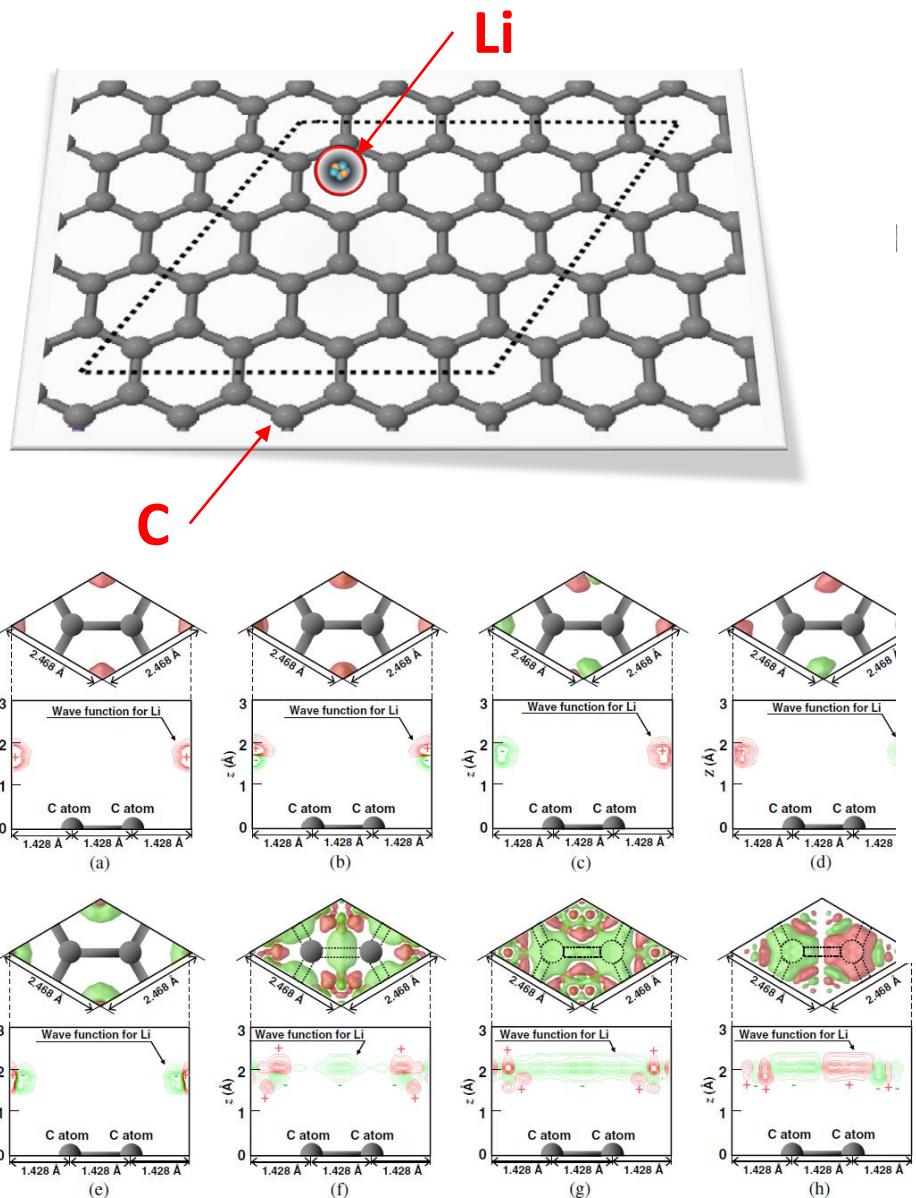


Quantum adsorbed energy of hydrogen: **-0.58eV**

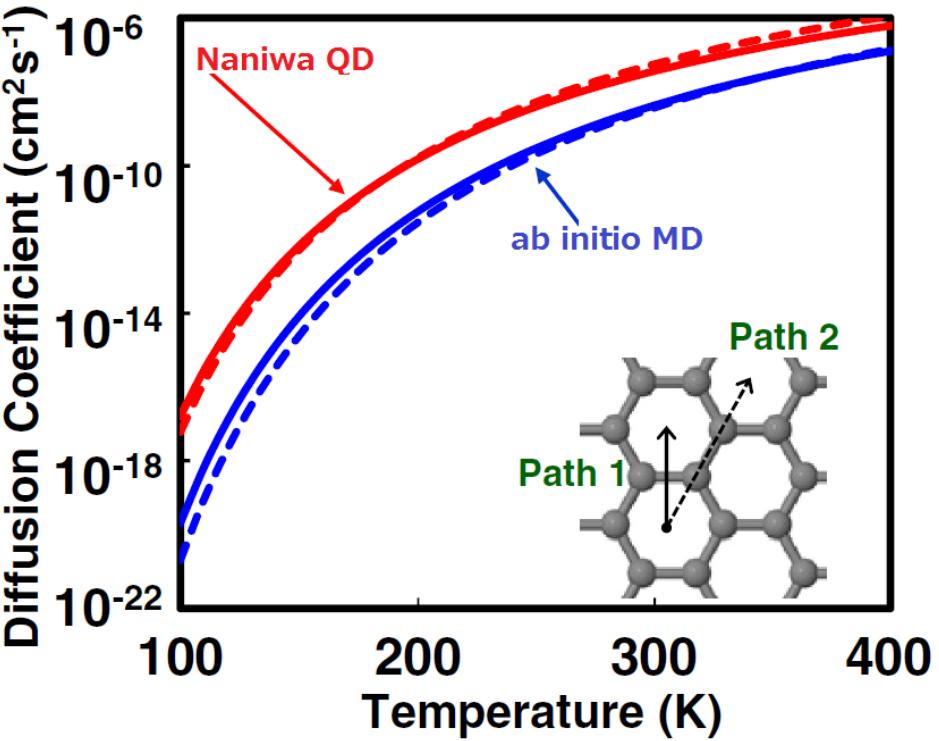
TDS experimental data : **-0.59 eV or -0.65eV**

X. Zhao, et al., J. Chem. Phys. 124(2006)194704

Li on graphene



Negative electrode of Lithium-ion battery



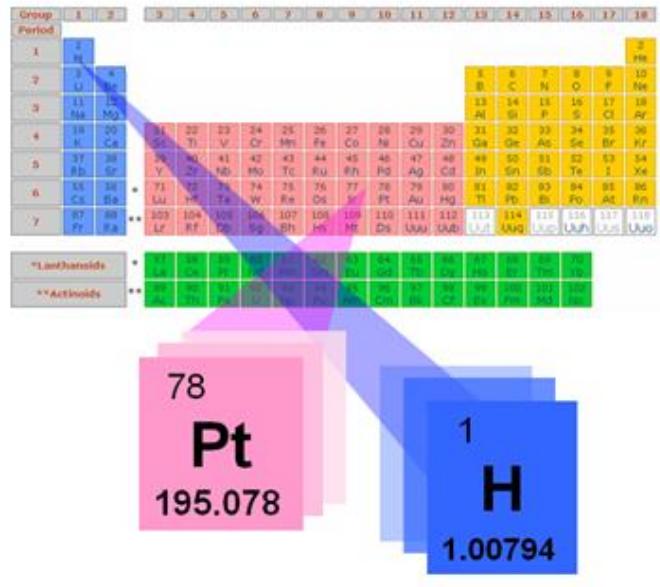
Activation barrier: $E_a(\text{path 1}) < E_a(\text{path 2})$

Our quantum simulation scheme: Naniwa

Interactions between nucleus is calculated by DFT based first principle calculations

... (*)

Potential energy
for nucleus motions: $U_n(\mathbf{R})$



Solve the Schrödinger
equation for nucleus
motion

... (**)

Wave function for
nucleus motion

Derive the
various
physical
quantities



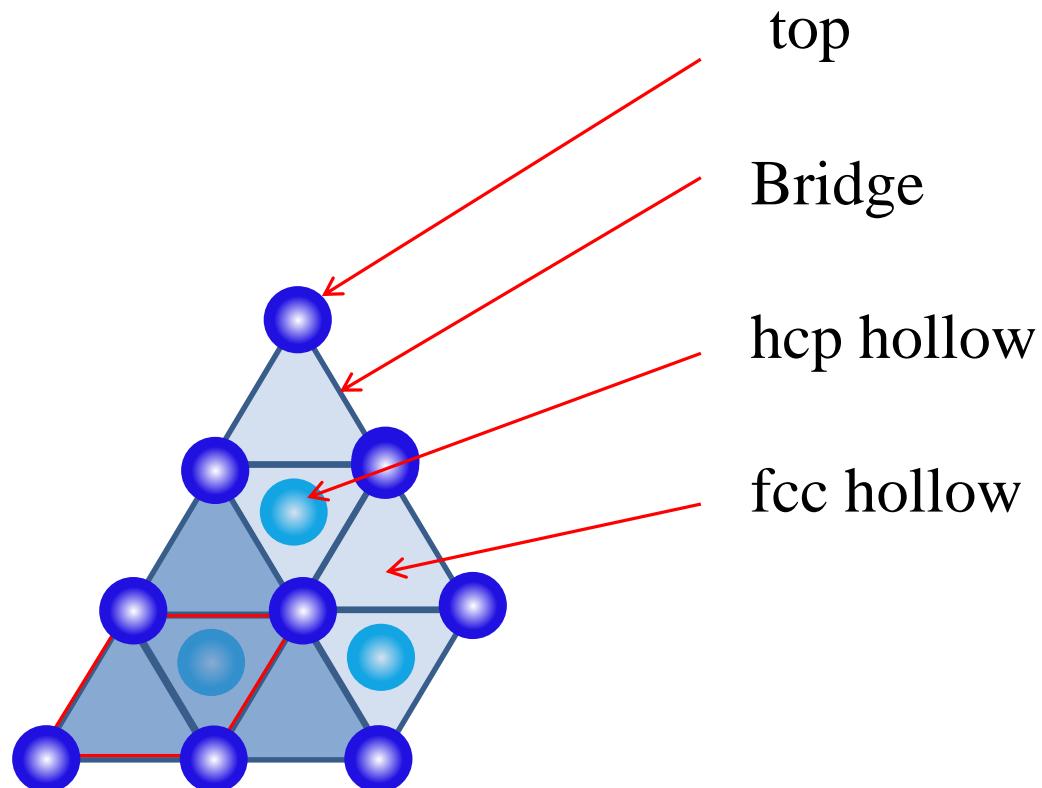
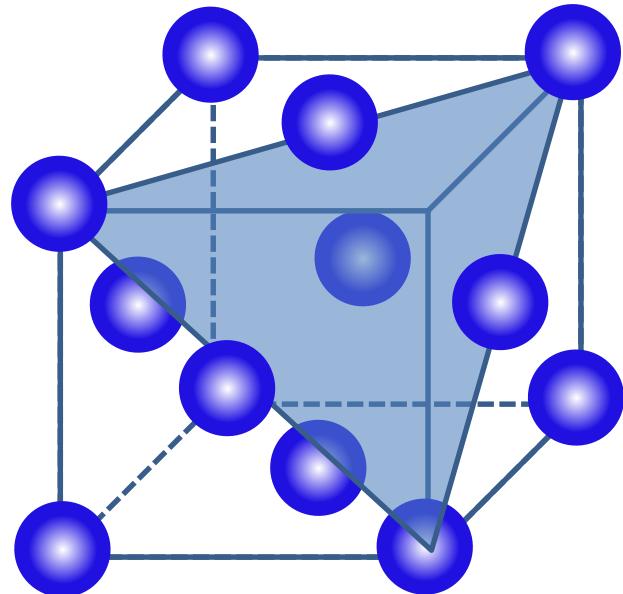
-Parameters are only
atomic number of elements
-No fitting and no artificial procedure

- Some examples on (111) surface of fcc crystals

Quantum states of hydrogen atom motion
on the surface

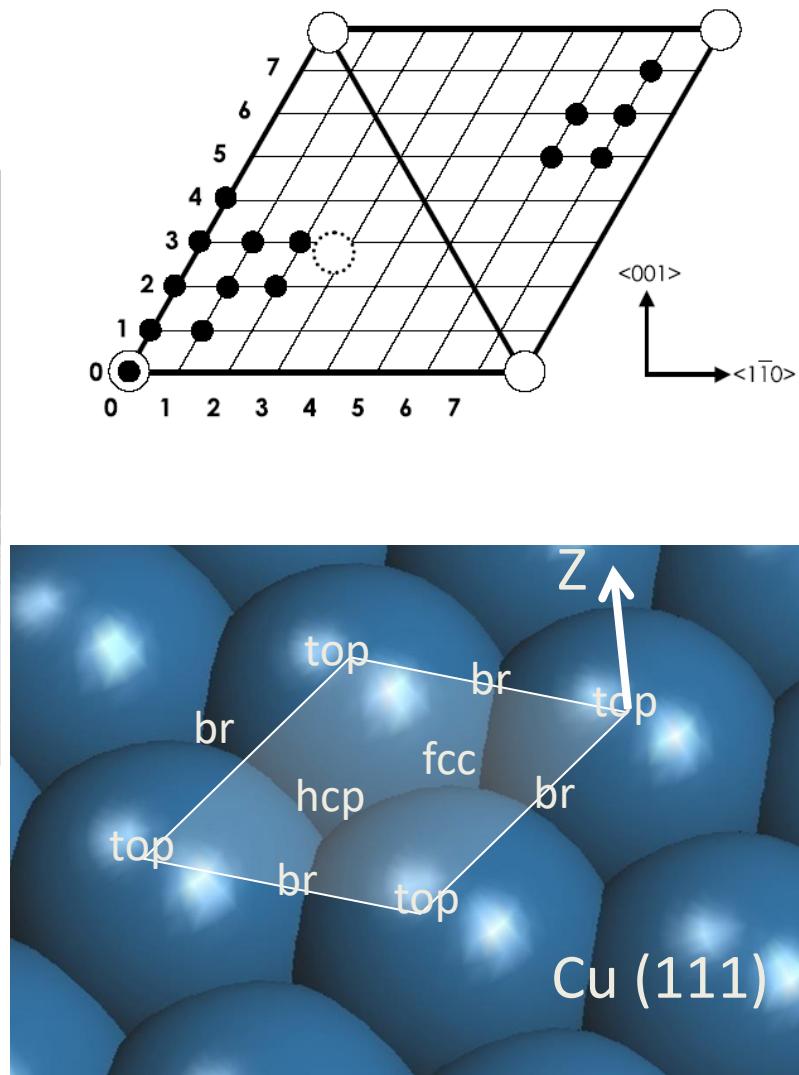
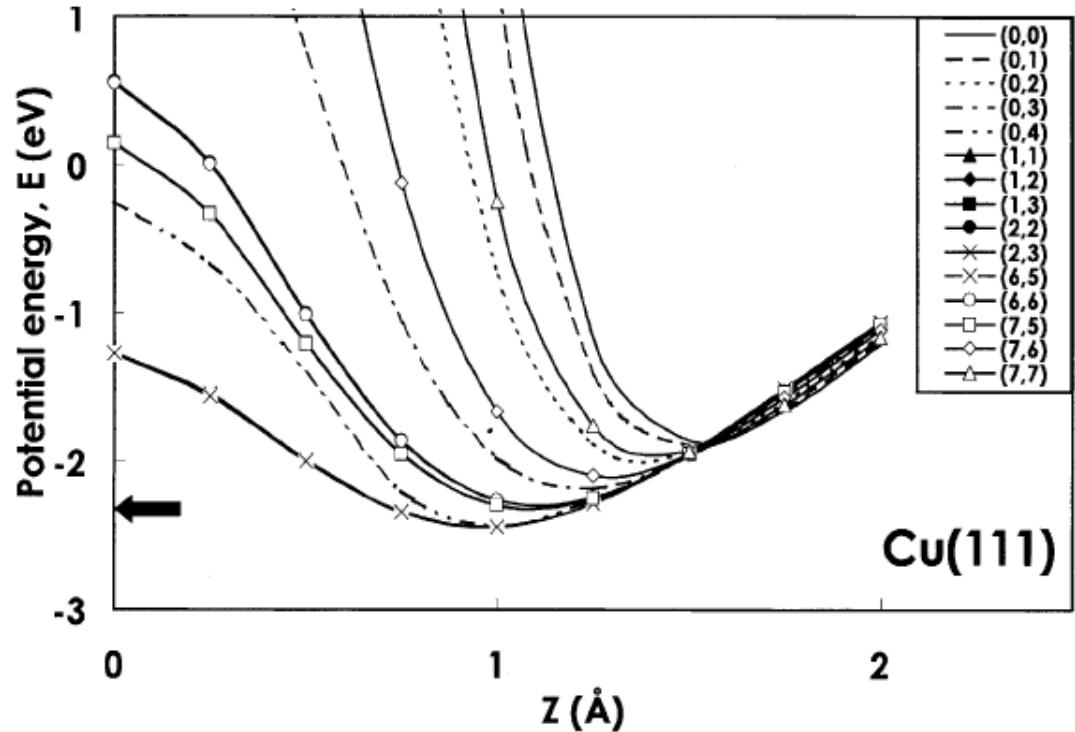
(111) surface of face center cubic crystals

(111) surface

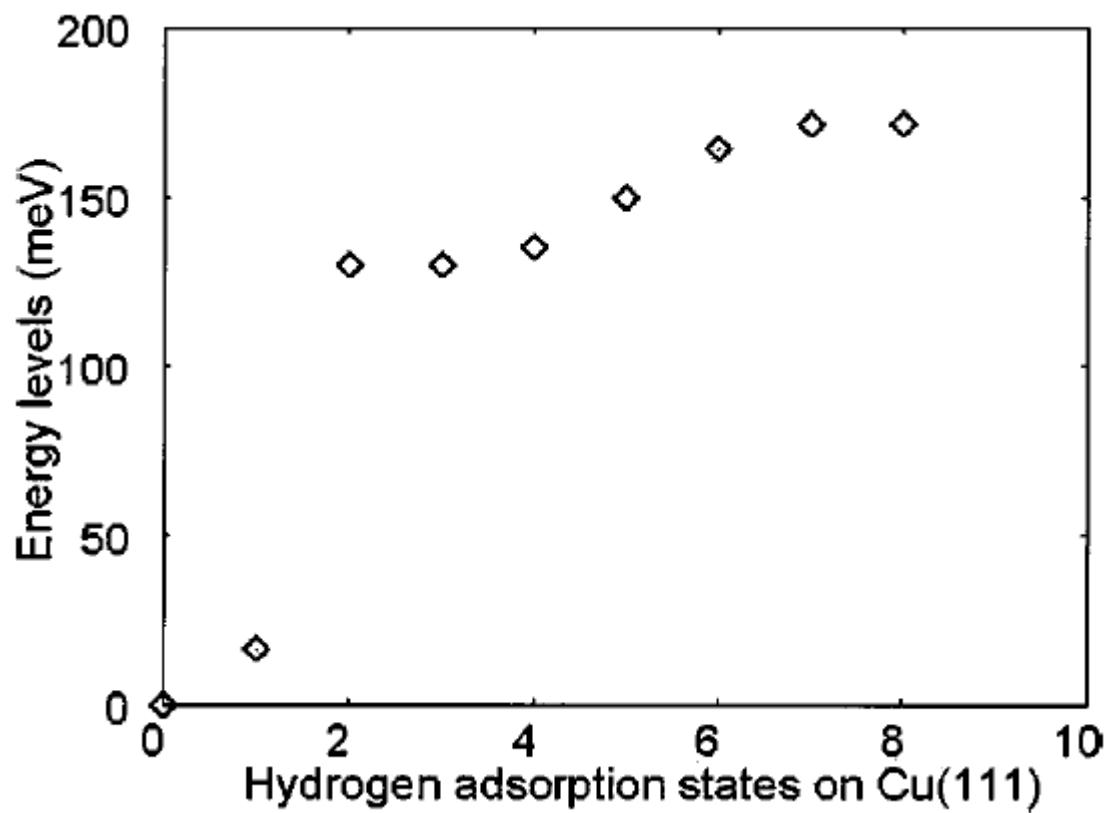


- First layer atom
- 2nd layer atom

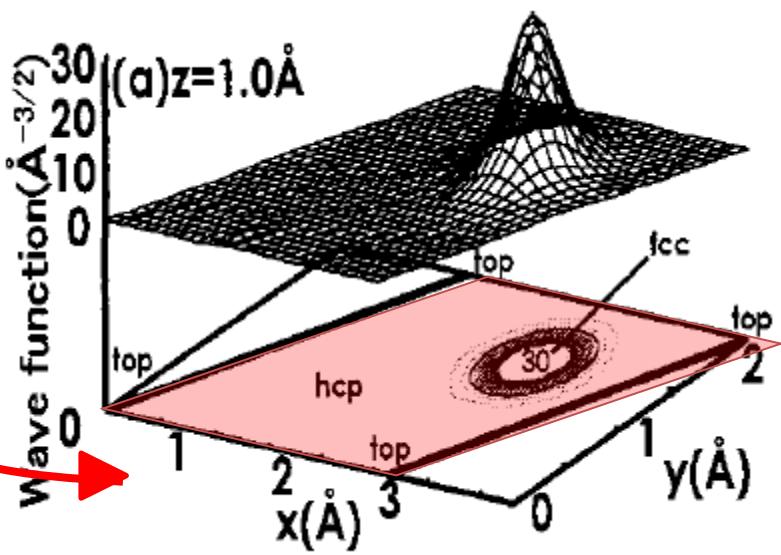
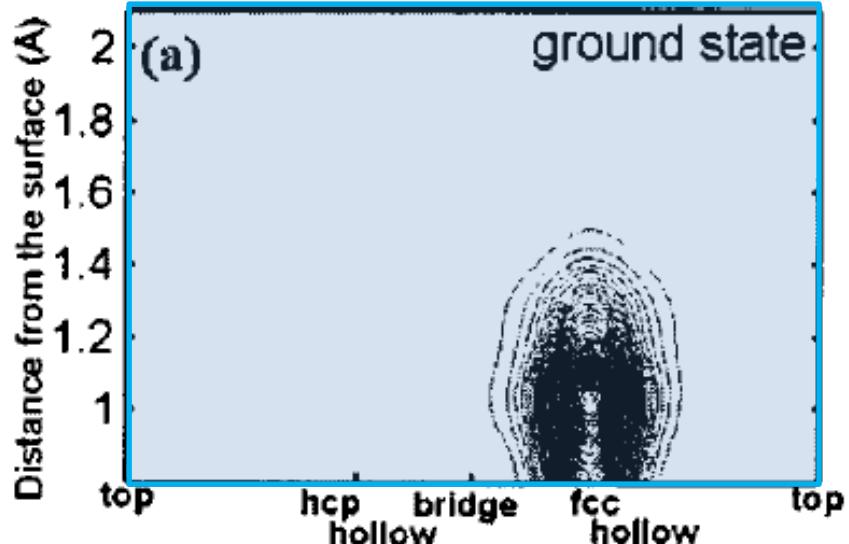
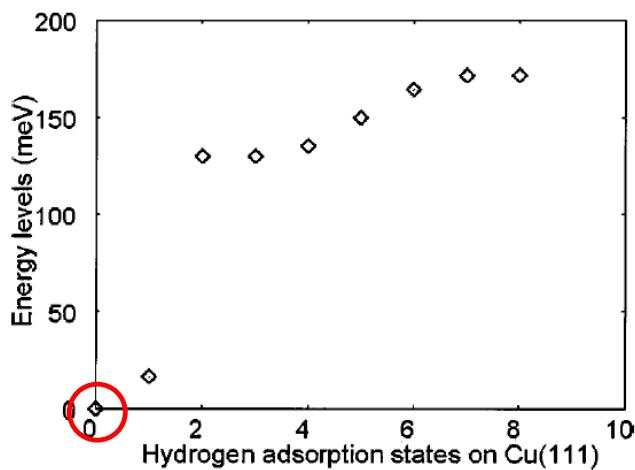
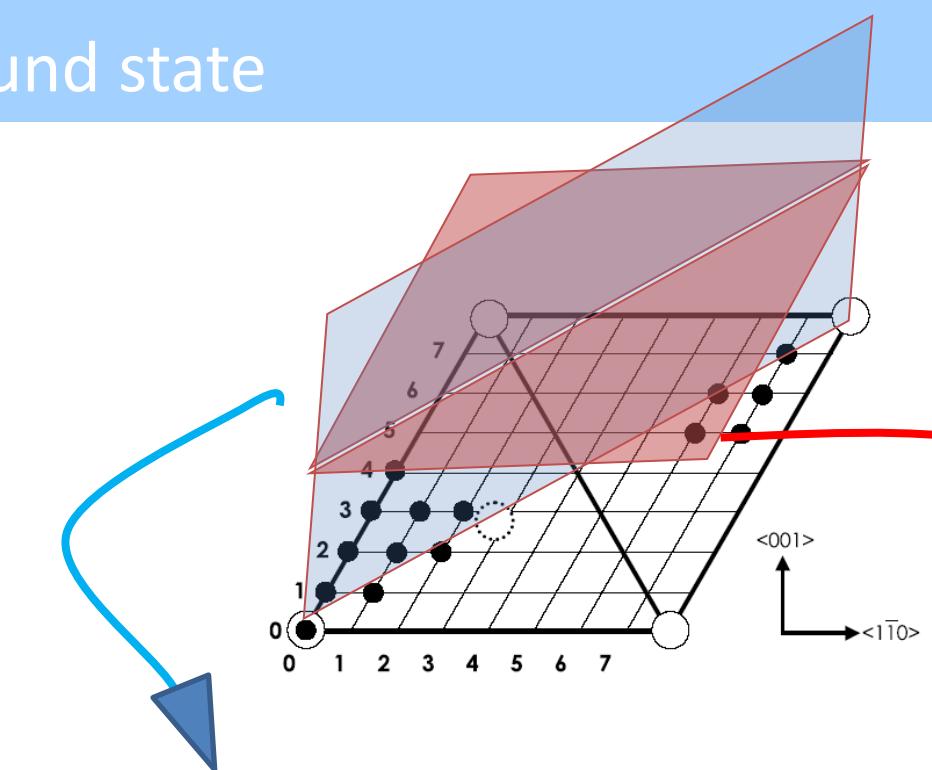
Potential energy surface of H motion on Cu(111)



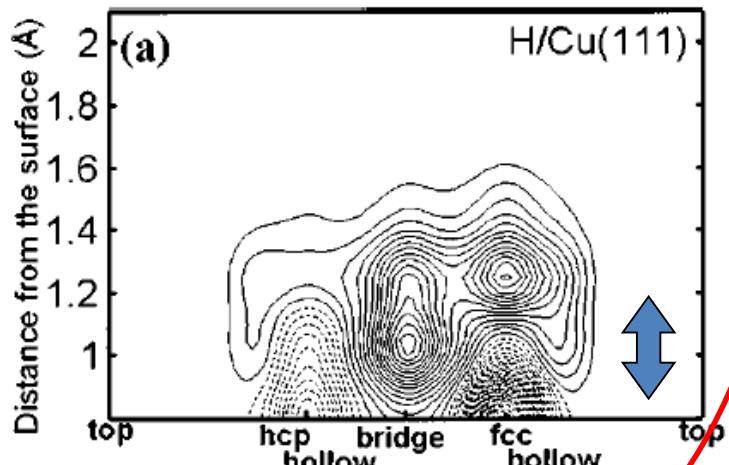
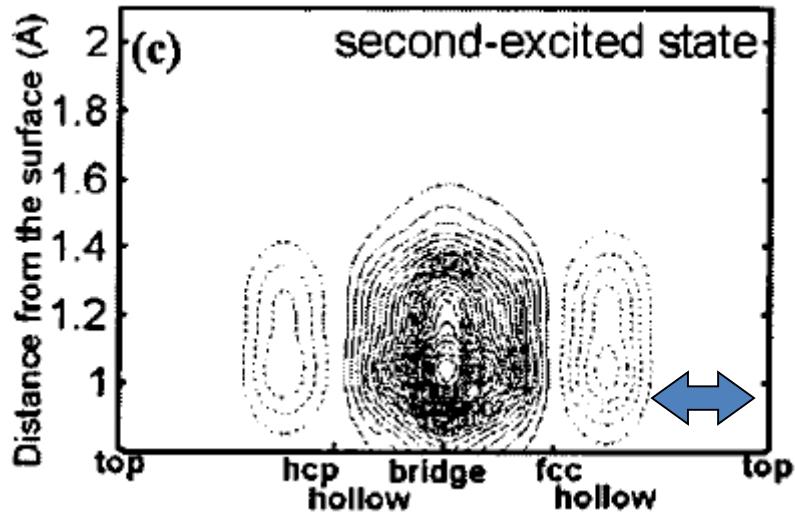
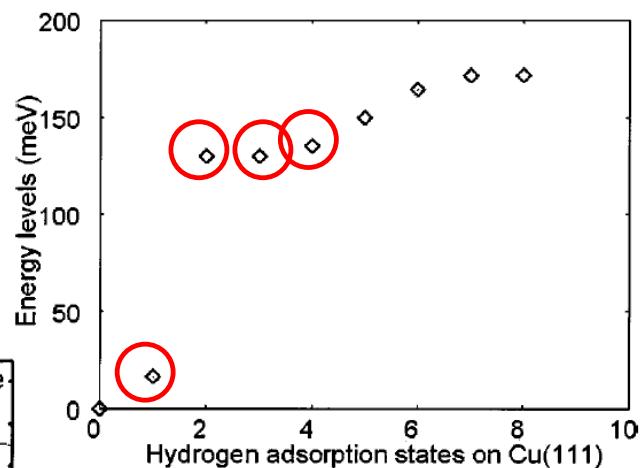
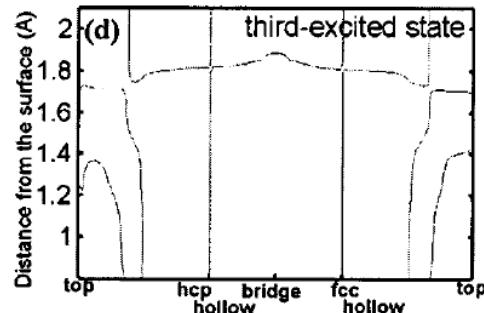
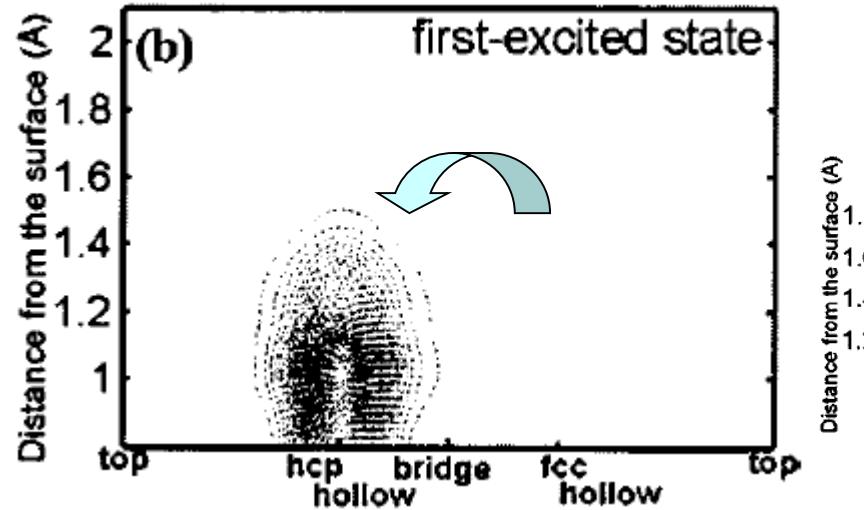
Eigenenergy for a H motion on Cu(111)



Ground state



The excited states

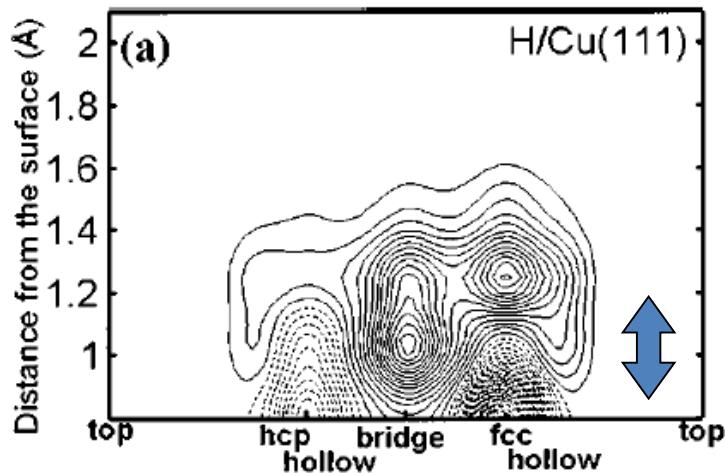
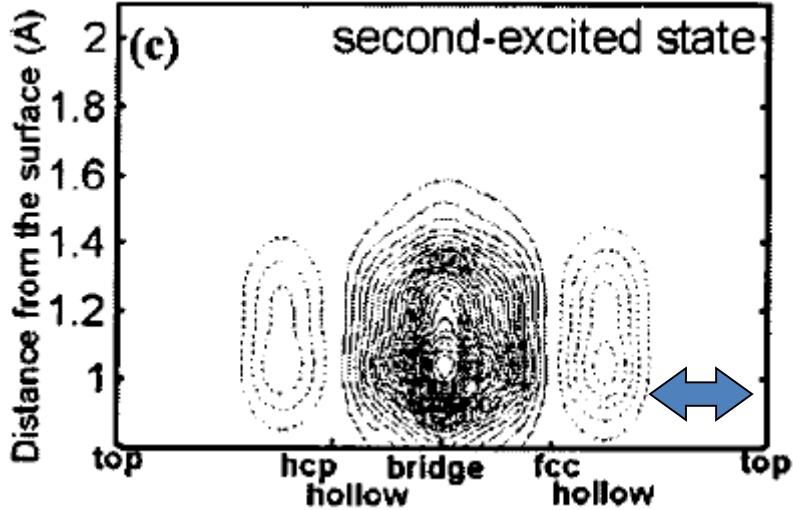
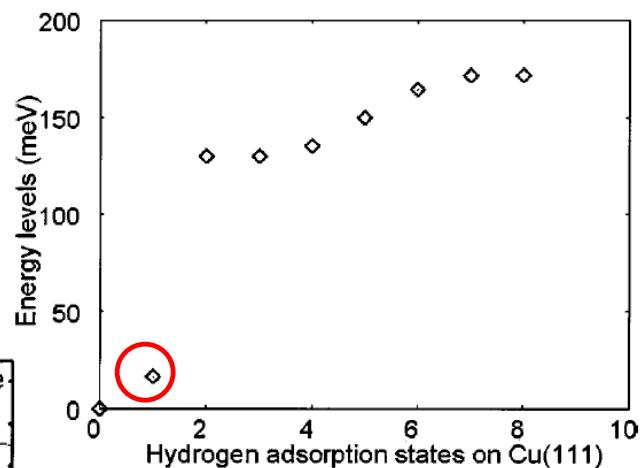
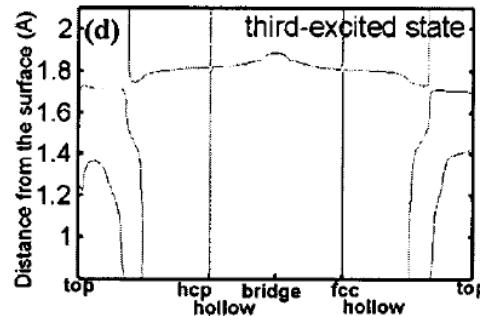
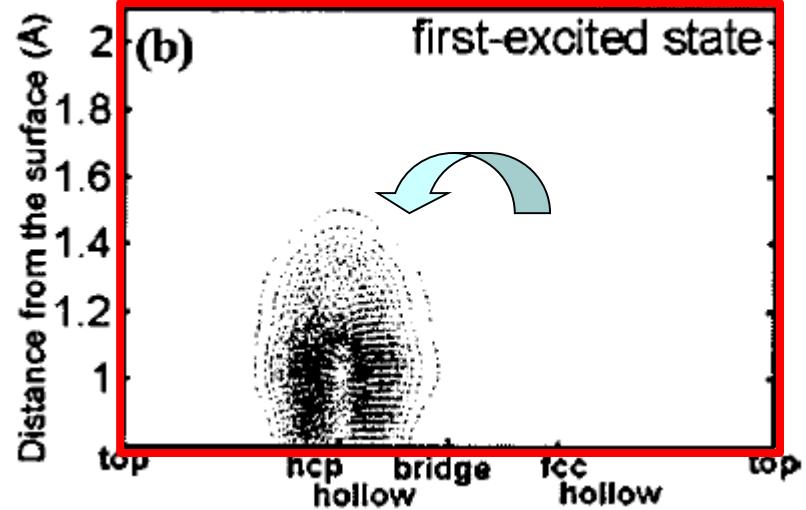


Exp. 129meV by HREELS

39

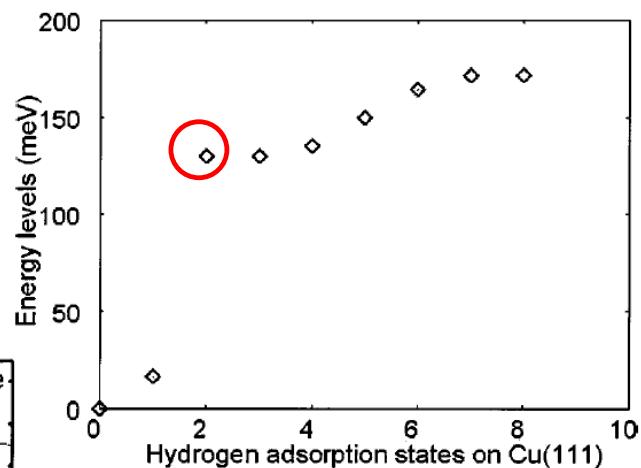
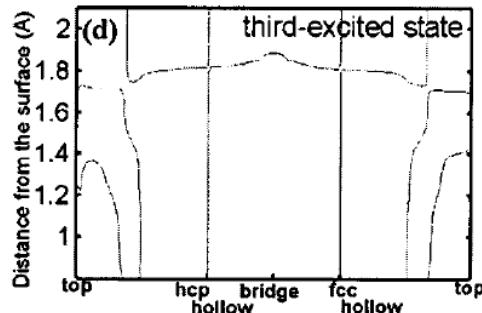
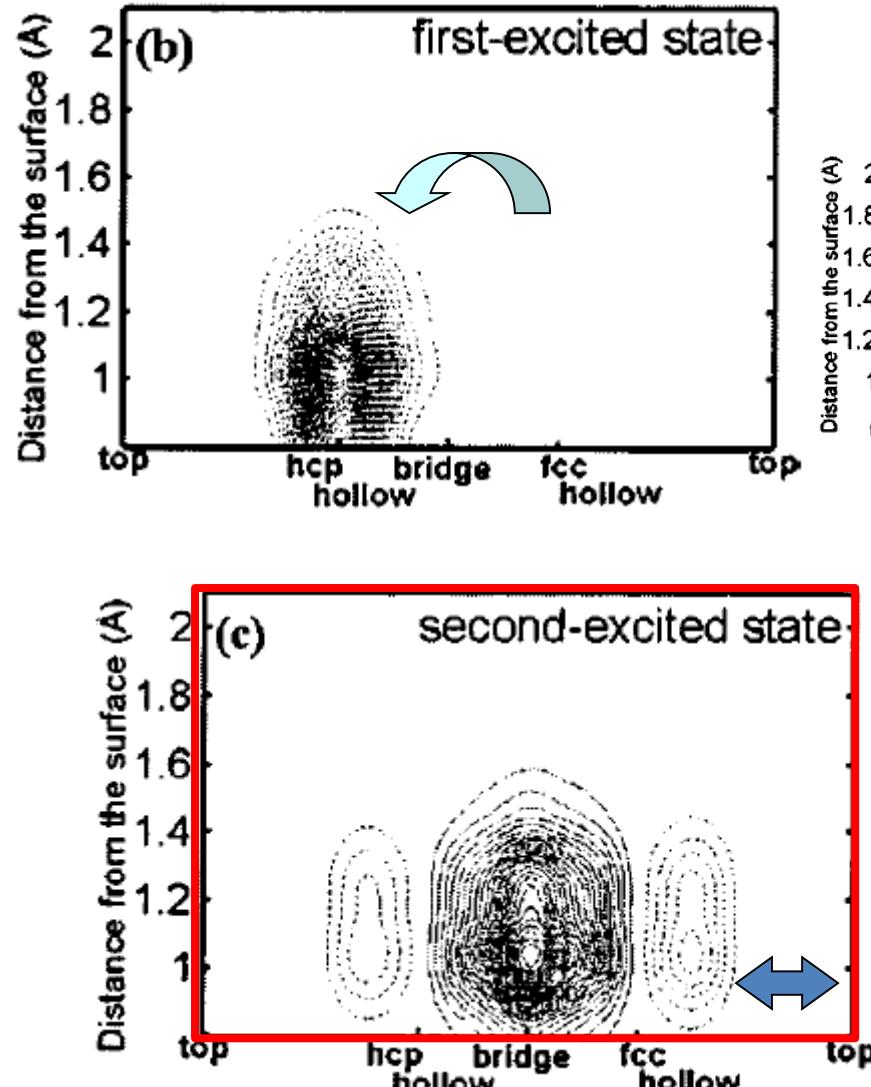
G.Lee et al., Surf. Sci. 498(2002)229.

The excited states

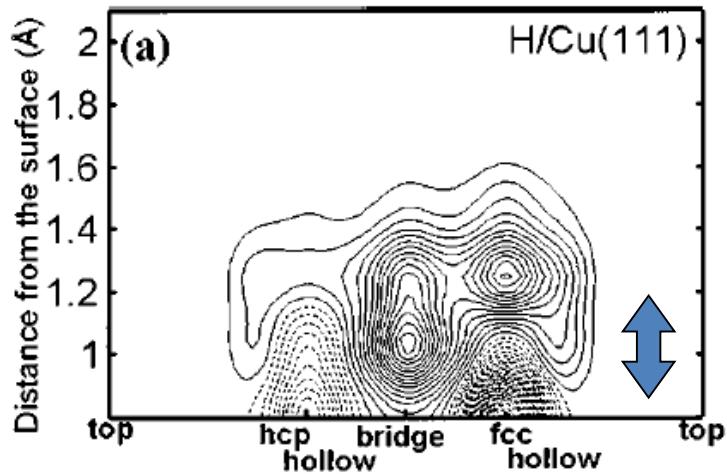


Exp. 129meV by HREELS

The excited states

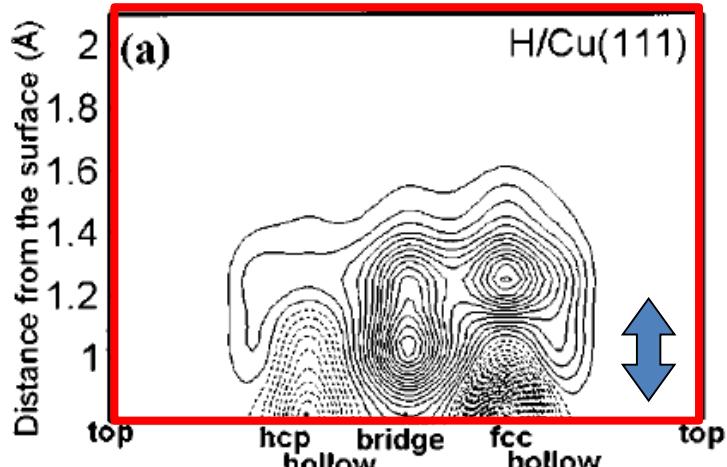
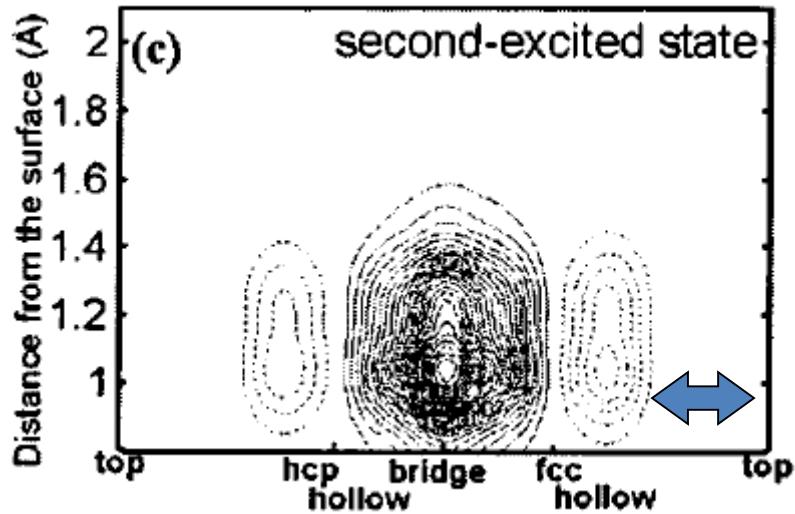
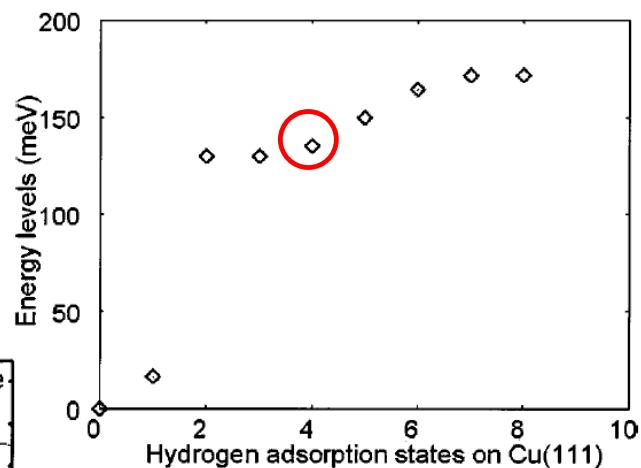
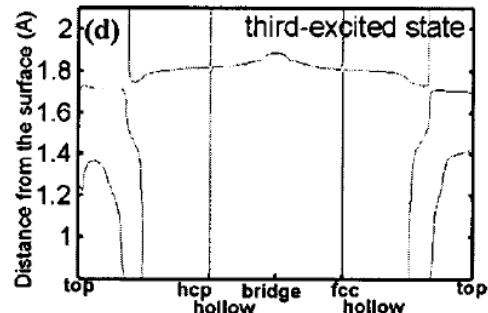
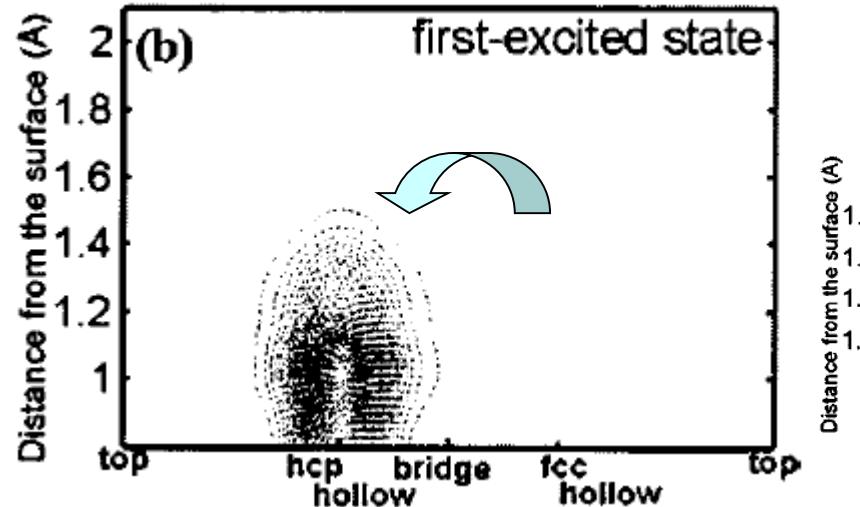


$$E_4 - E_0 = 135 \text{ meV}$$



Exp. 129meV by HREELS

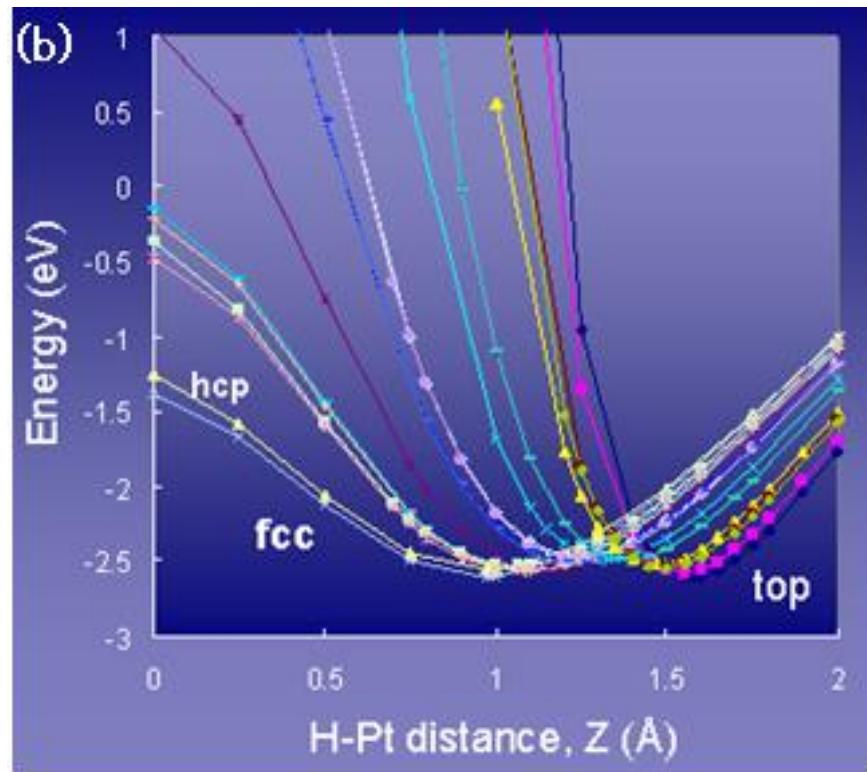
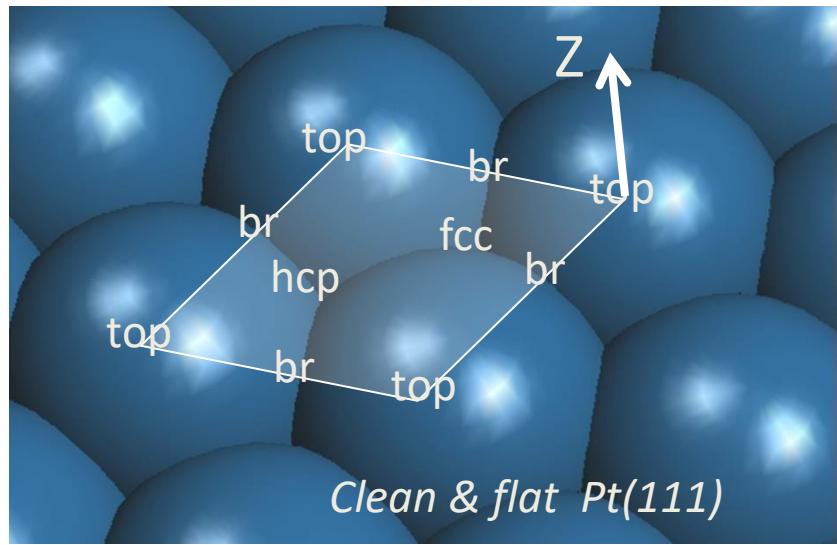
The excited states



Exp. 129meV by HREELS

G.Lee et al., Surf. Sci. 498(2002)229.

Potential energy surface of H motion on Pt(111)



H atom adsorption energy well depths :

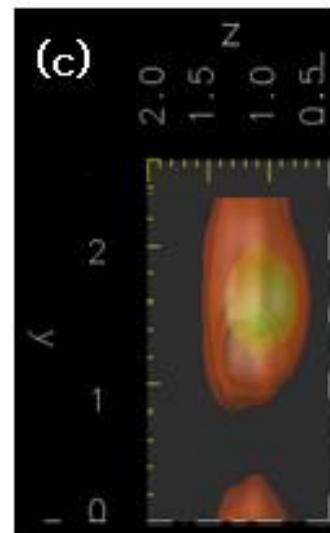
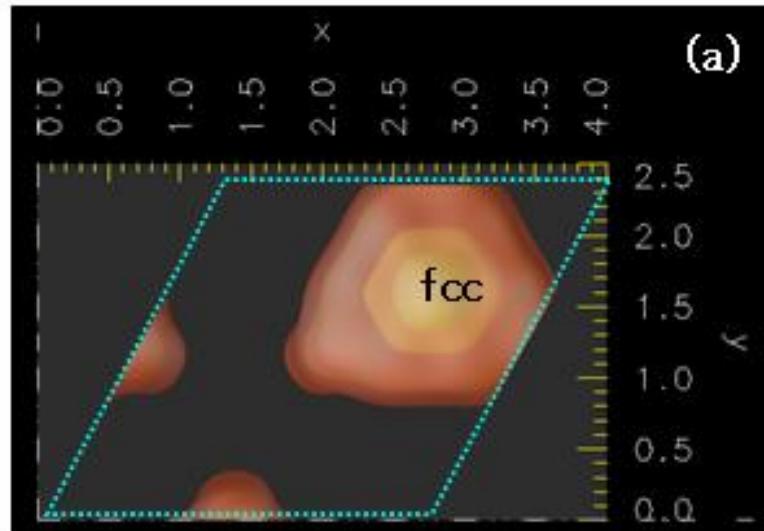
Top : 2.609 eV, fcc hollow : 2.607 eV, hcp hollow : 2.562 eV



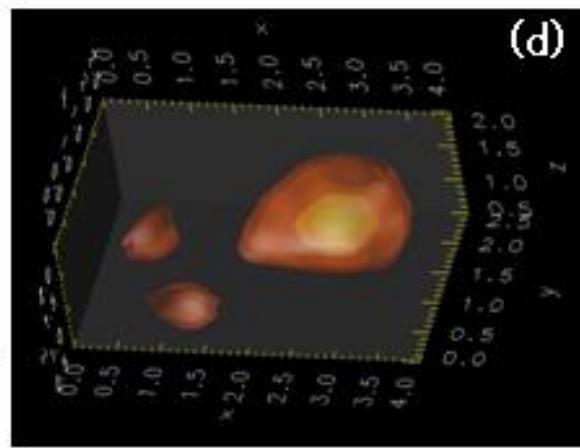
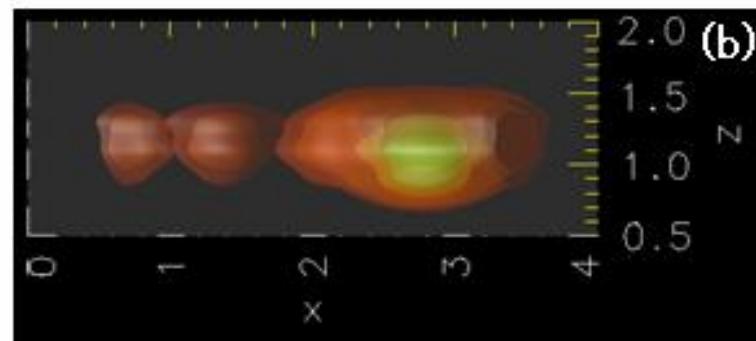
Most stable adsorbed site is On-top site !?

Quantum mechanical effects for hydrogen motions

Ground state of hydrogen motion on Pt(111) surface



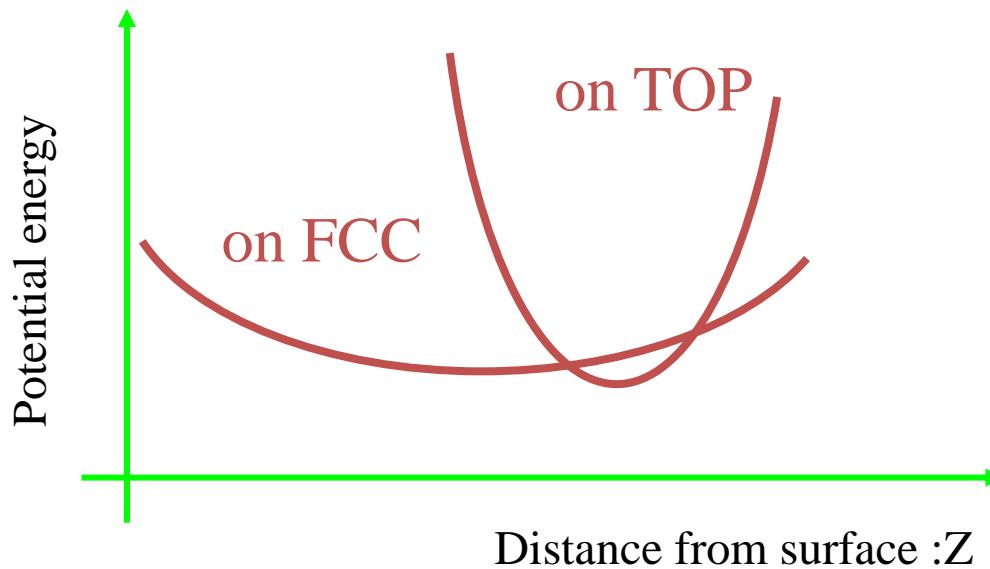
Ground state wave
function: $\Psi_0(x,y,z)$ 。
Eigen energy: -2.461eV



Å

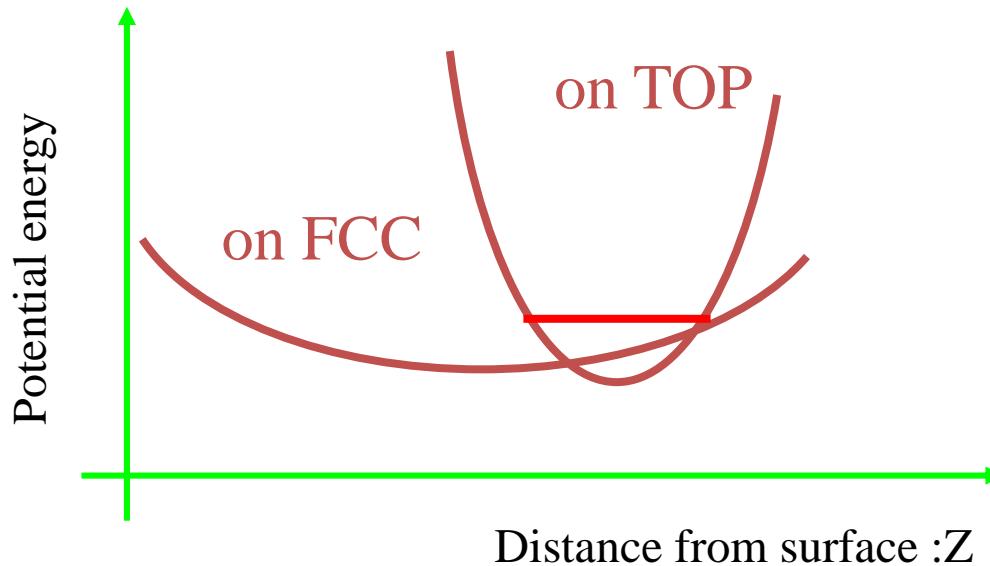
Why ground state is localized around fcc hollow site?

Potential energy : top site < fcc hollow site



Why ground state is localized around fcc hollow site?

Potential energy : top site < fcc hollow site



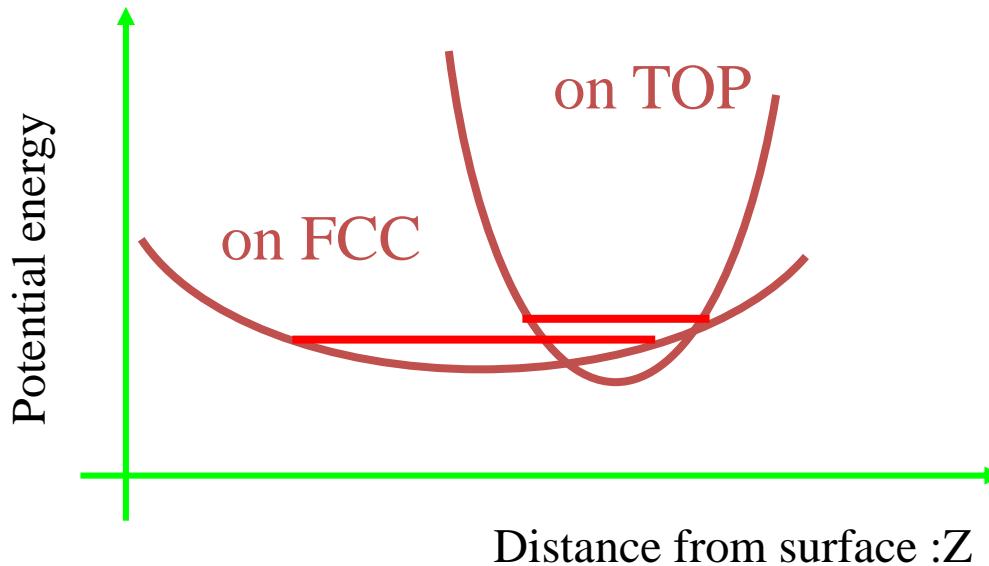
Heisenberg uncertainty principle

Narrower space confinement causes the higher kinetic energy.

Quantum mechanical effects

Why ground state is localized around fcc hollow site?

Potential energy : top site < fcc hollow site



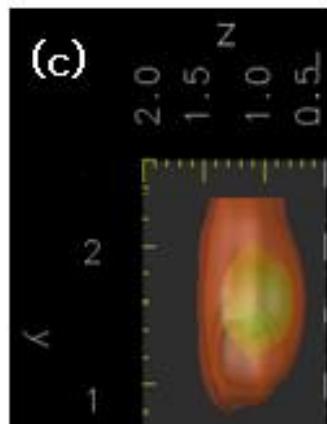
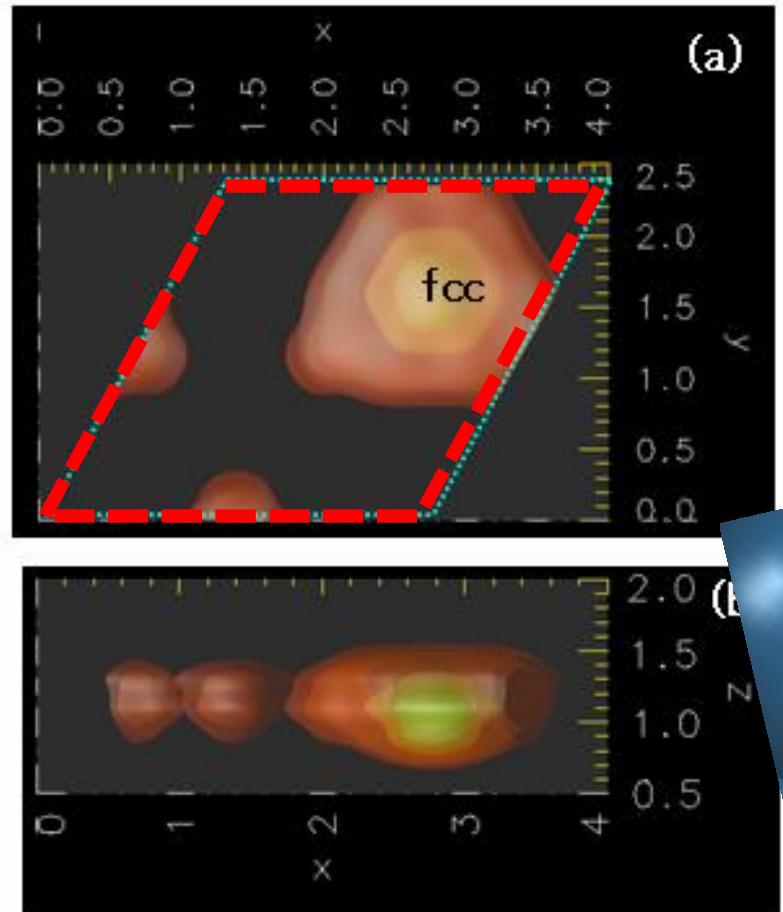
Heisenberg uncertainty principle

Narrower space confinement causes the higher kinetic energy.

Quantum mechanical effects

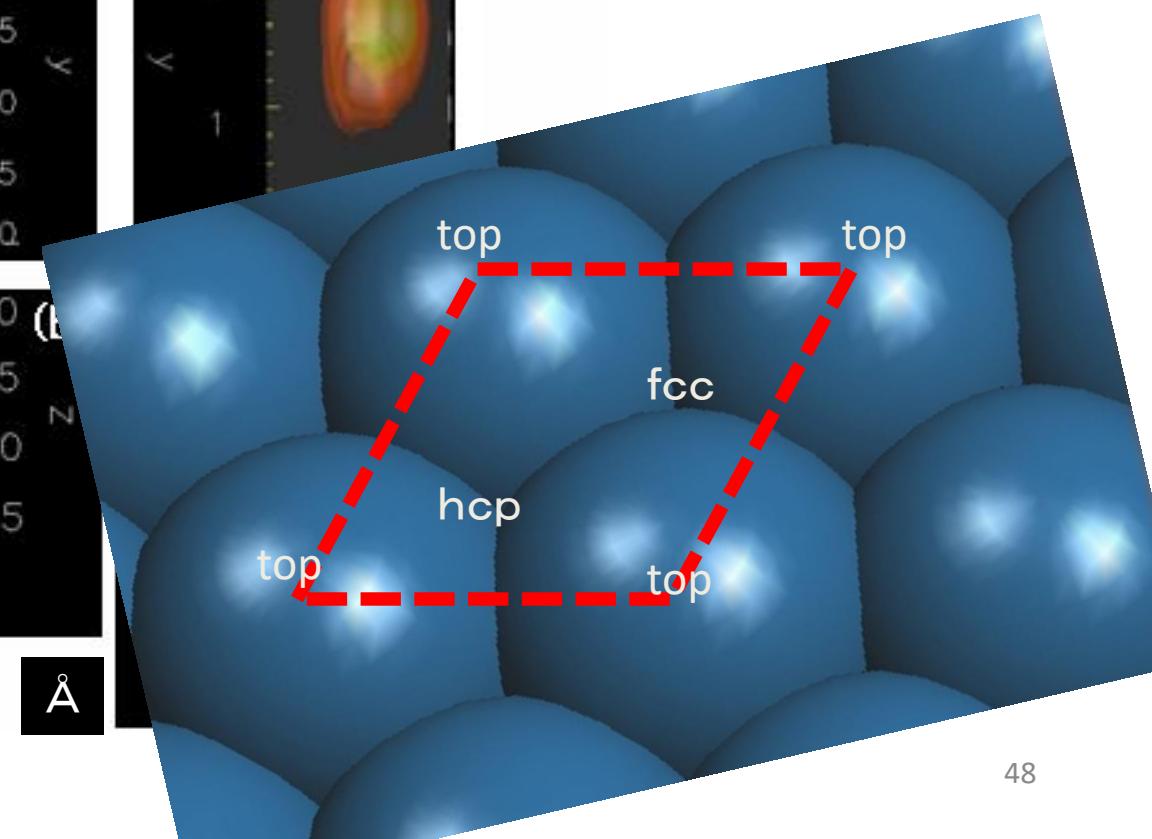
Quantum mechanical effects for hydrogen motions

Ground state of hydrogen motion on Pt(111) surface



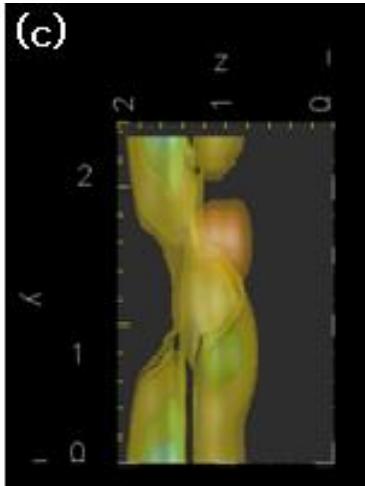
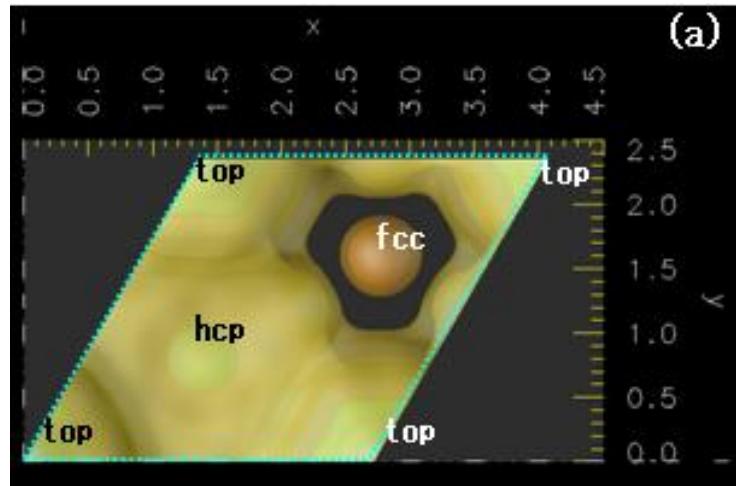
(c)

Ground state wave
function: $\Psi_0(x,y,z)$.
Eigen energy: -2.461eV



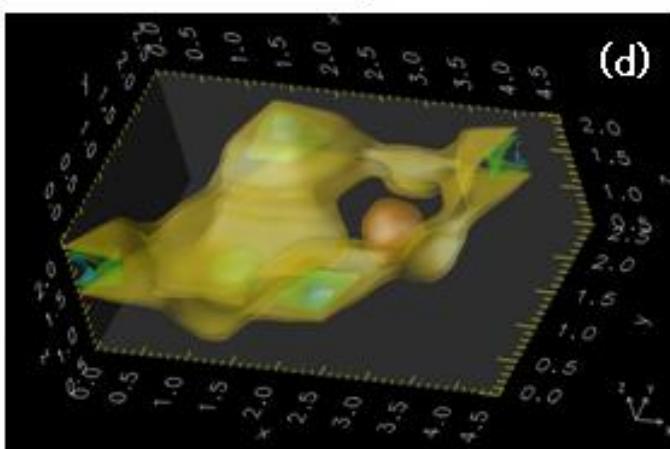
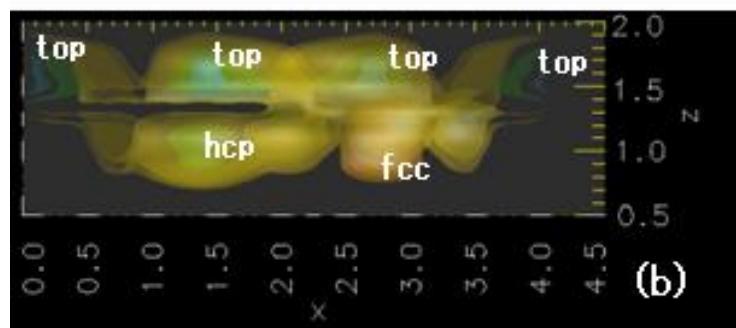
As for migration of H atom on the electrode surface

The 1st excited state wave function expands on the surface



The first excited state wave function: $\Psi_1(x,y,z)$.

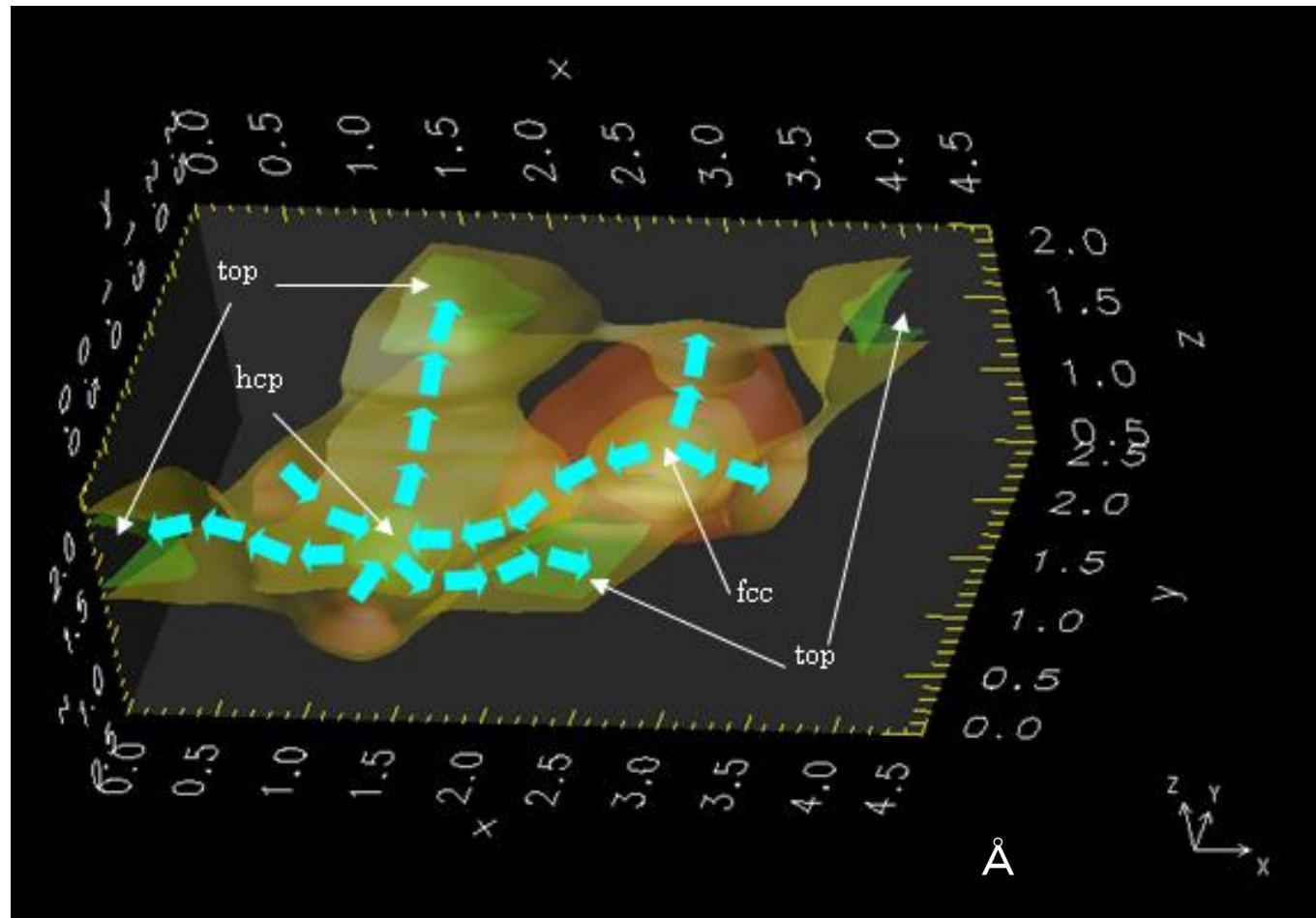
Eigenenergy: -2.431eV



Å

Energy difference between first and ground state:

$$E_1 - E_0 = 30 \text{ meV}$$



Diffusion can occur at typical fuel cell operational temperature !

Naniwa codes

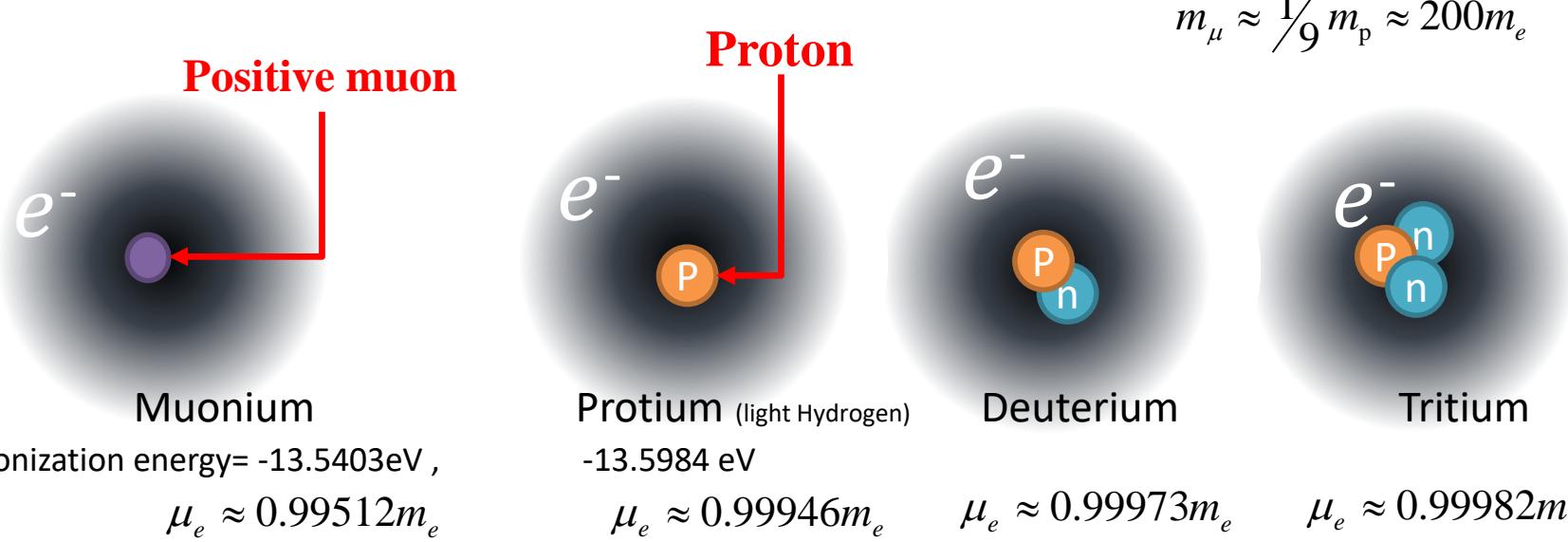
- Positive muon is also one of our target quantum particles as hydrogen isotope, which is useful as a magnetic probe in the materials, mu-SR.

Name	Anti muon (Positive muon)
Particle statistics	Fermionic
Mass	$105.658369(9) \text{ MeV}/c^2$
Electric charge	$+e$
Spin	$\frac{1}{2}$
Mean lifetime	$2.19703(4) \times 10^{-6} \text{ sec}$

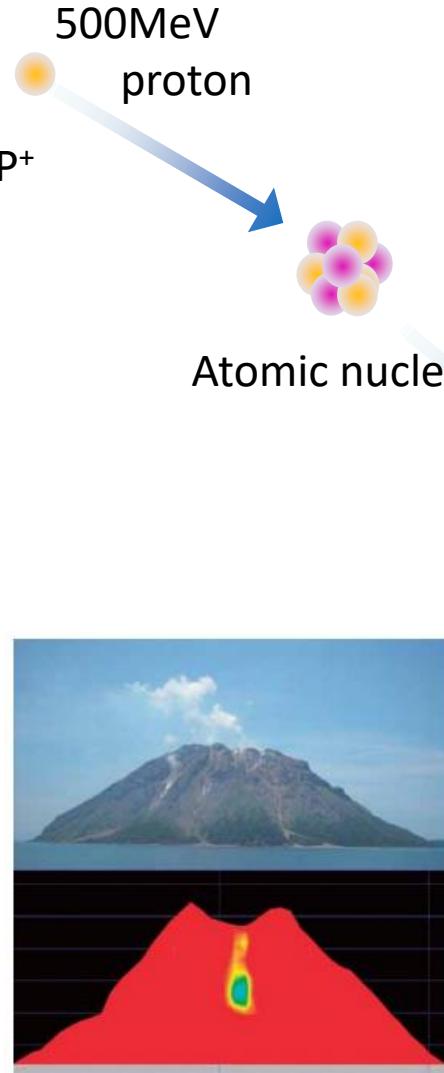
$$m_\mu \approx \frac{1}{9} m_p \approx 200 m_e$$

$$m_p > m_\mu \gg m_e$$

$$m_\mu \approx \frac{1}{9} m_p \approx 200 m_e$$



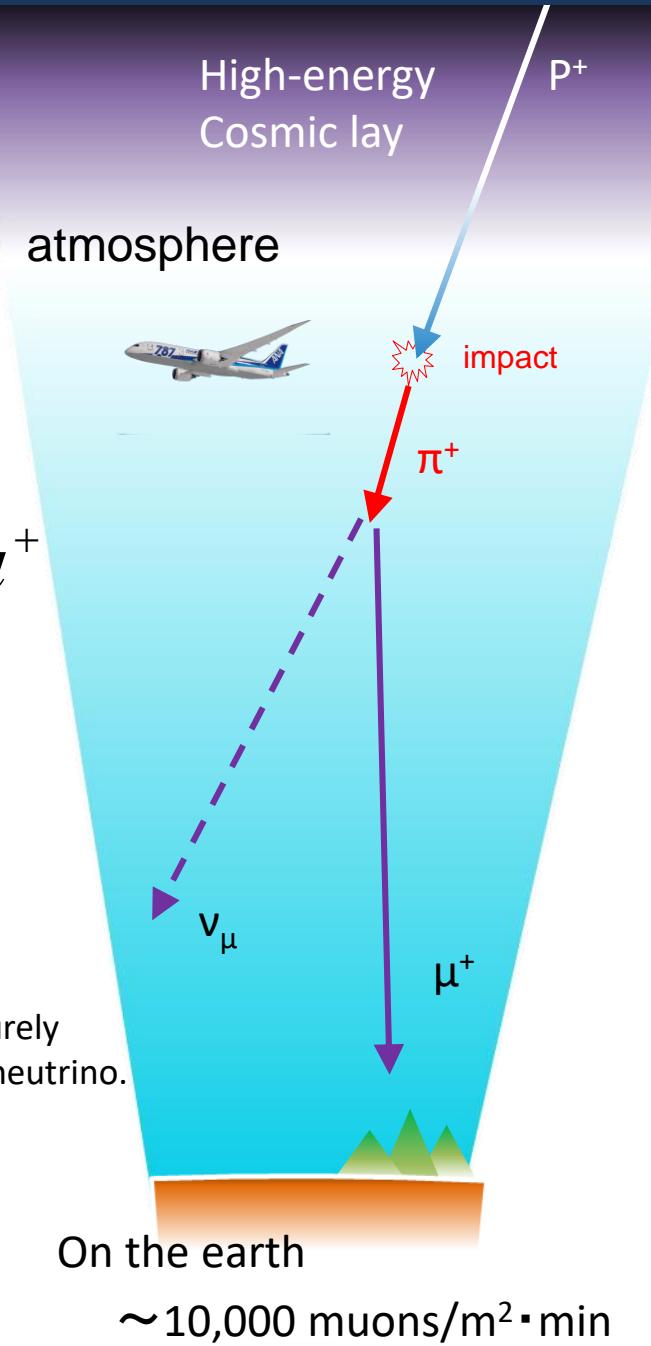
How to get the positive-muon ?



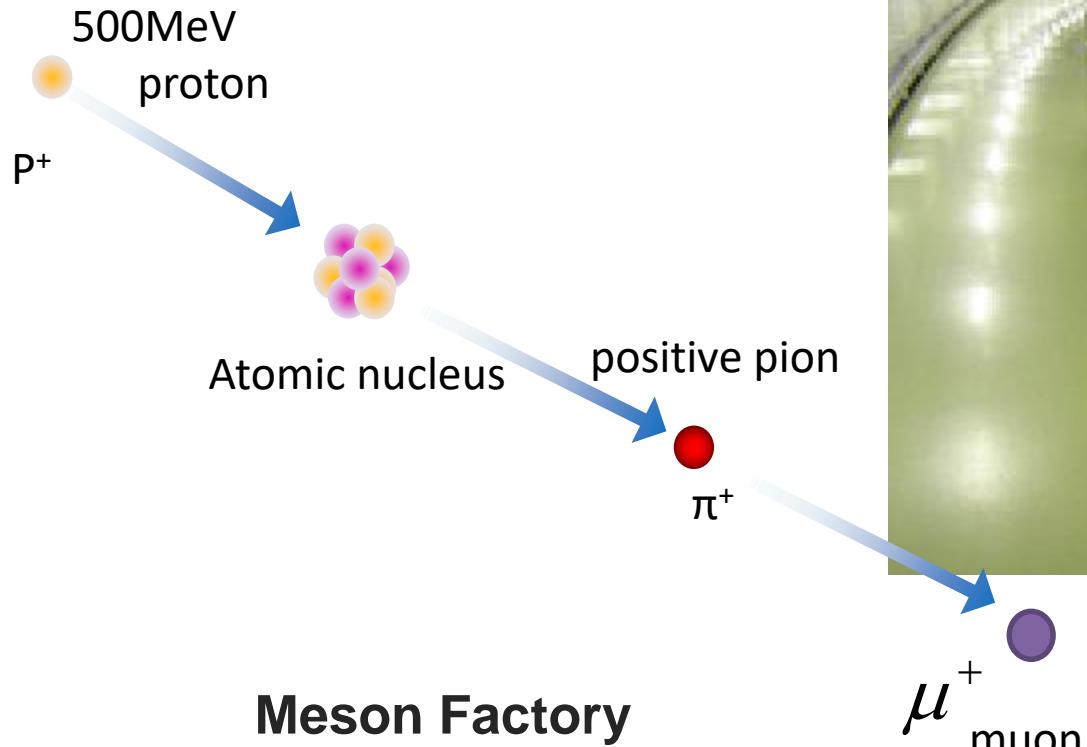
Magma image in volcano by muon

H.Tanaka, Butsuri (in Japanese) 65 (2010) pp. 70.

The primary decay mode of a pion is a purely leptonic decay into a muon and a muon neutrino.



How to create the positive-muon ?



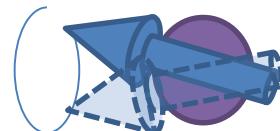
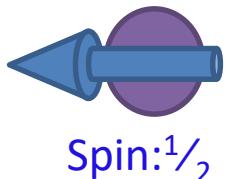
Japan Proton Accelerator Research Complex

20 muons/ (ϕ 3mm spot) · sec

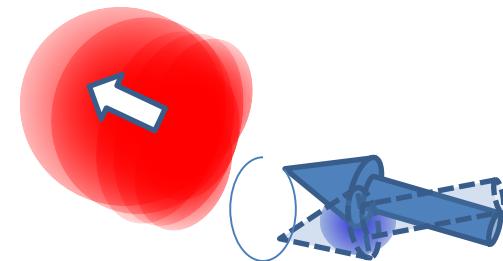
$\sim 2,000,000$ muons/ $m^2 \cdot min$



(Anti) muon μ^+



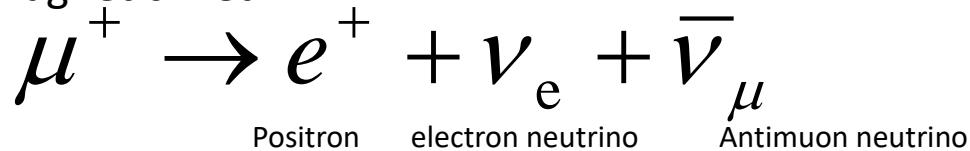
positron emission



the created muon is 100% spin polarized

Muon spin spectroscopy : μ SR

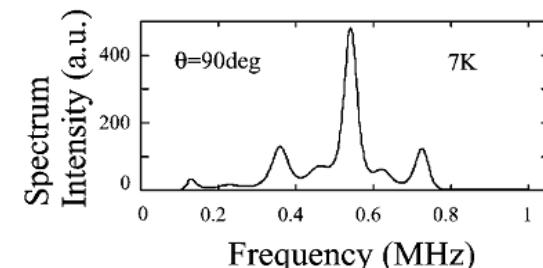
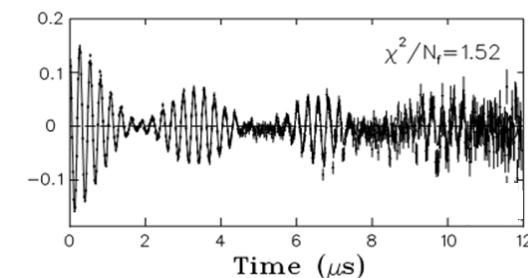
precession movement
in the magnetic field



Life time : $\tau \sim 2.2 \mu\text{sec}$

$$\text{Larmor precession frequency: } f = \frac{\gamma}{2\pi} B$$

$$\text{gyromagnetic ratio of muon: } \frac{\gamma}{2\pi} = 135.53 \text{ MHz/T}$$



μ SR muon spin rotation method

Gyromagnetic ratio : 135.53 MHz/T

We can measure the magnetic field around muon in materials

Effective Magnetic interaction =

Hyperfine interaction between muon spin and electron spin.

Muon

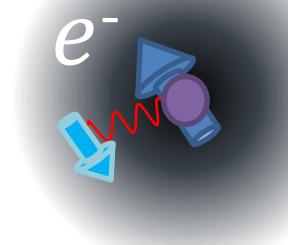


Hyperfine
interaction:

gyromagnetic
ratio: 135.53 MHz/T

Muonium

= muon + electron



4463 MHz

13940 MHz/T

We can also measure the occupation number of hydrogen isotope impurity in materials.

Muon or Muonium
Positive hydrogen ion(proton) or Hydrogen atom

Muon in the Si crystal

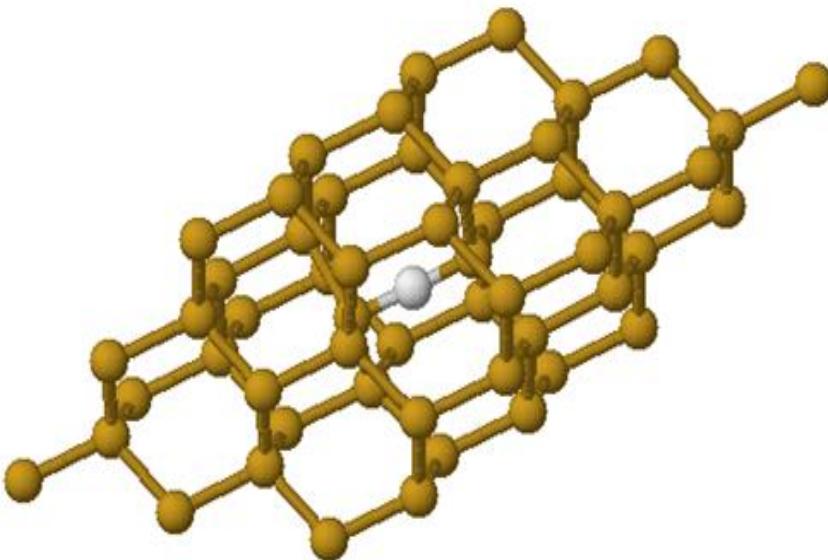
Experimental results of μ SR suggests two type of muon site.

hyperfine constant

BC-site (Bond center): very low frequency \rightarrow shallow donors

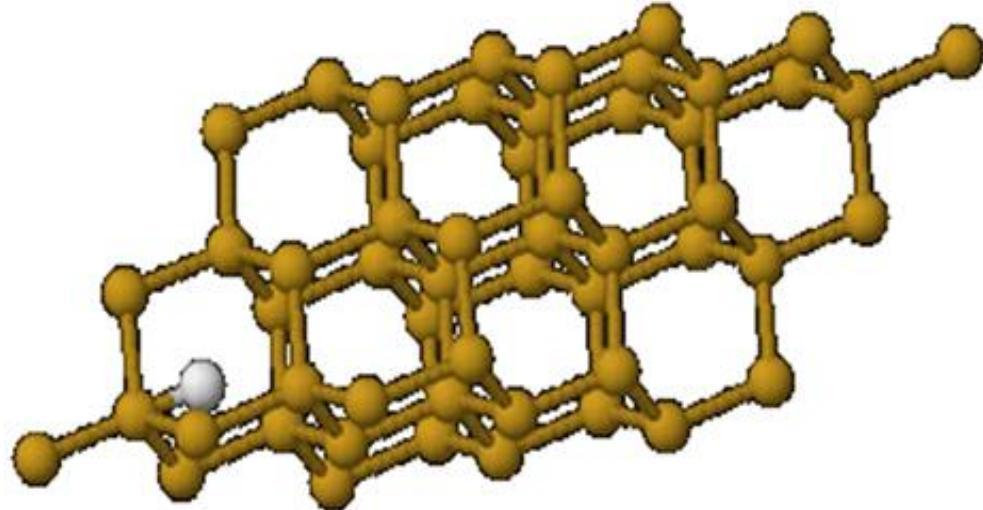
T-site (Cage center): 45 % of a free Muonium $A_0 = 4.46\text{GHz}$

S. F. J. Cox, Rep. Prog. Phys. 72 (2009) 116501.



BC-site

3x3x3
54 Si atoms



T-site

Muon in the Si crystal

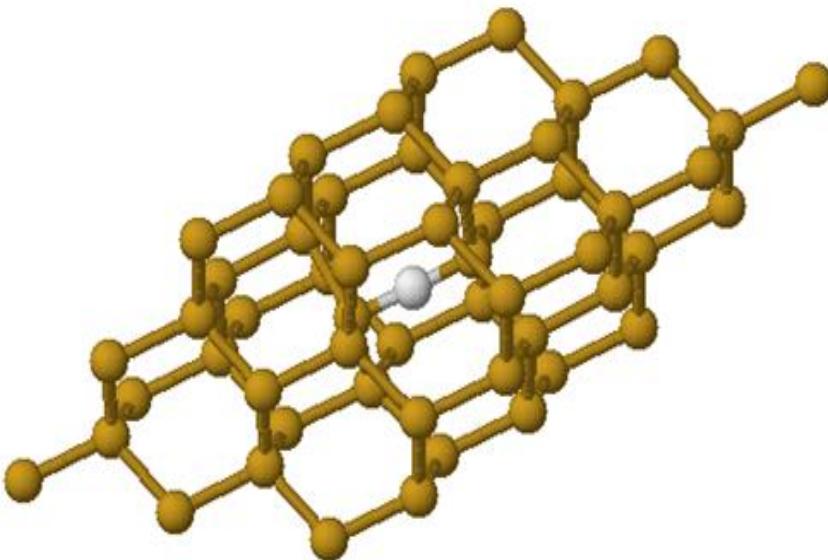
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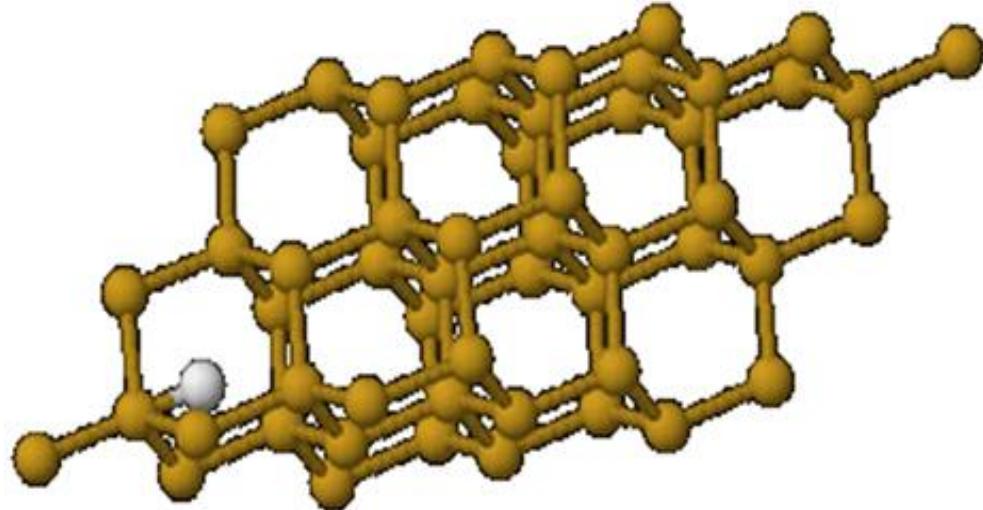
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S. F. J. Cox, Rep. Prog. Phys. 72 (2009) 116501.



BC-site

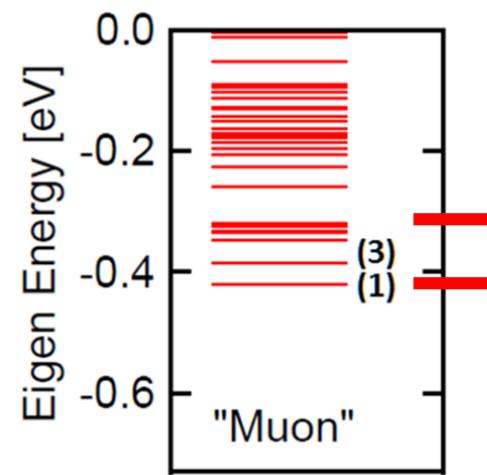
3x3x3
54 Si atoms



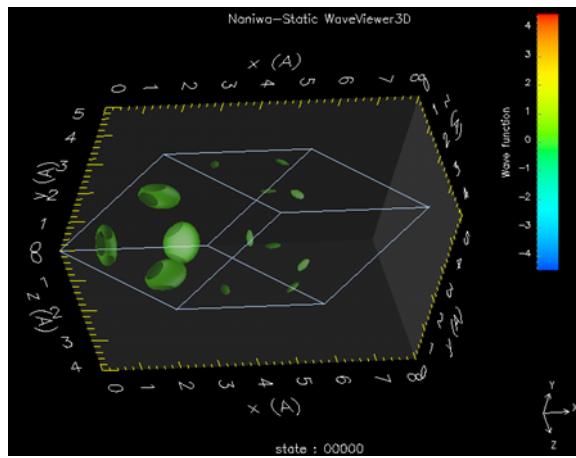
T-site

Naniwa results

diamond structure of Si



Ground state : BC-site

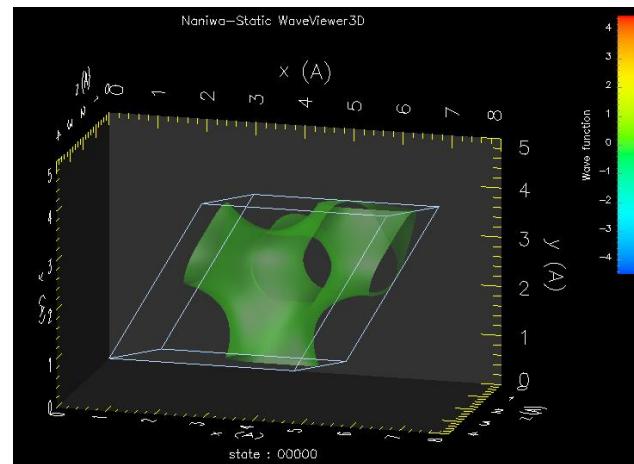


Calculated hyperfine constant

275 MHz

very low frequency

"shallow donor"

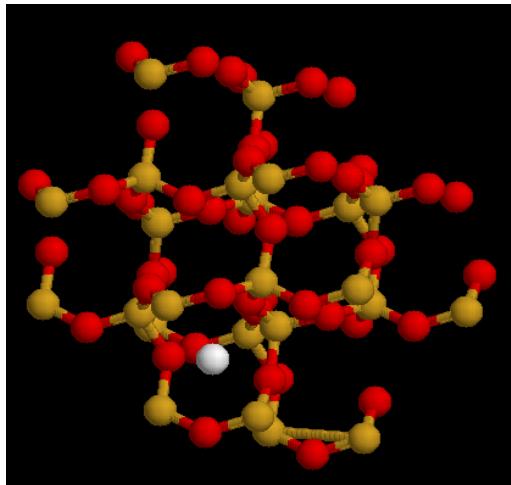


Excited state : T-site

1711 MHz

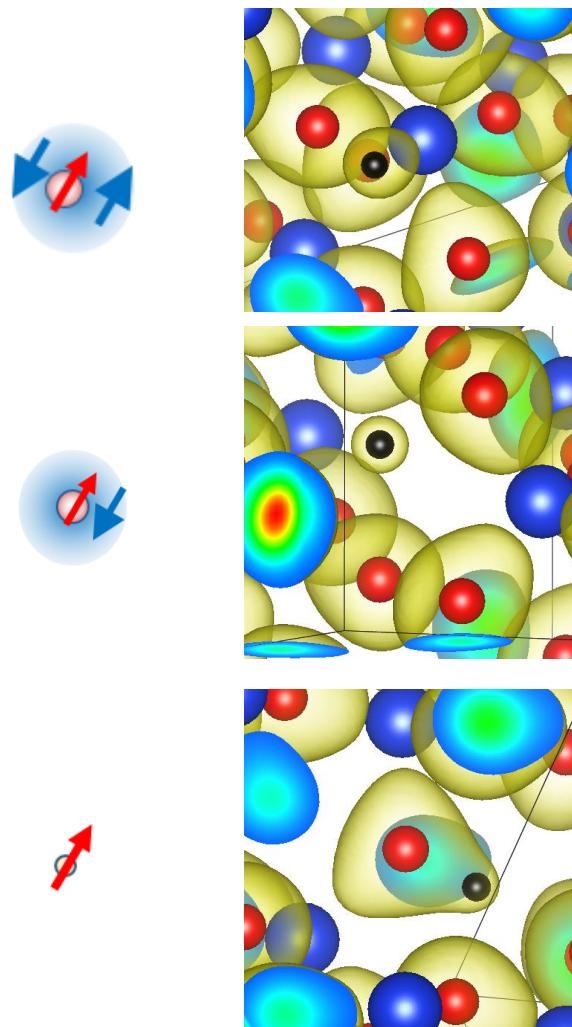
$\approx 45\%$ of A_0

Hydrogen isotope Muon in $\alpha\text{-SiO}_2$ Quartz



In the case of insulator and semiconductor,
the charged state of impurity may be changed by the its environment.

Hydrogen isotope in $\alpha\text{-SiO}_2$ Quartz



On-Si site

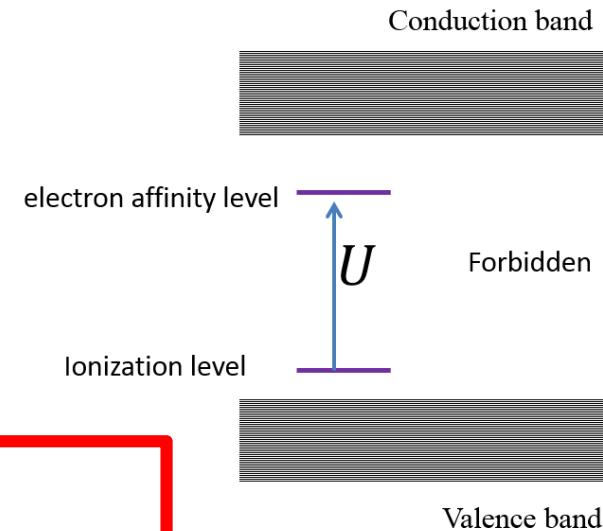
Si^+

In cage
of SiO_2

Vac.^0

On-O site

O^{2-}



μSR shows the value of
free Muonium $\approx A_0 = 4.46\text{GHz}$

Hydrogen isotope in $\alpha\text{-SiO}_2$ Quartz

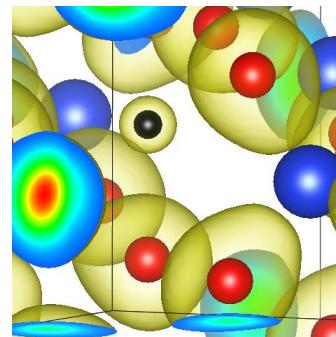
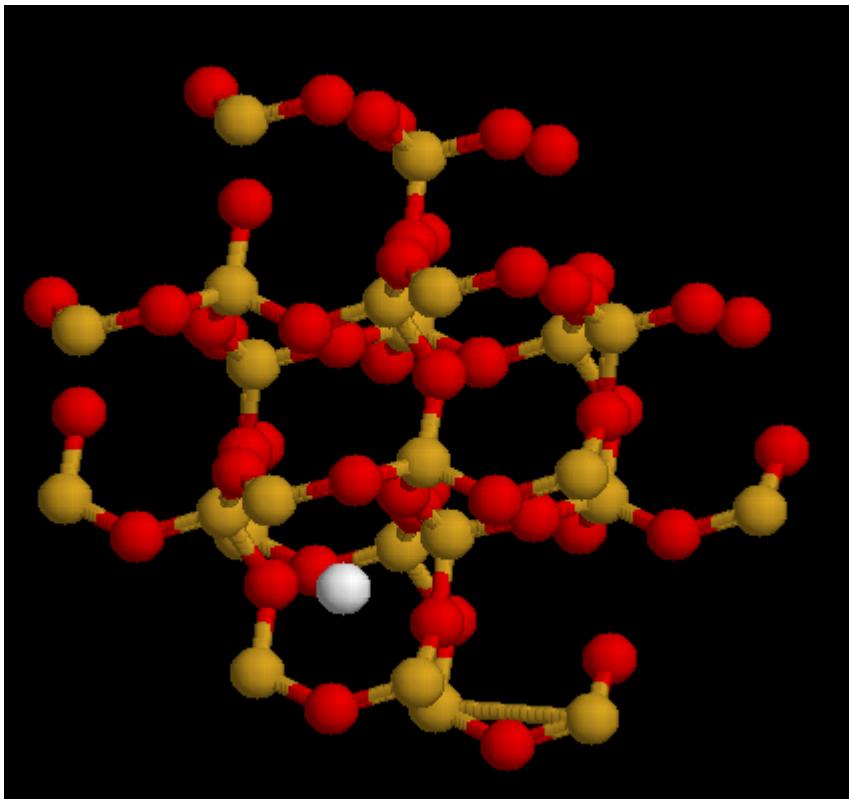
2x2x2 cell

Si: 24 atoms

O: 48 atoms

H: 1 atom

4.2% impurity hydrogen



In-cage site

Previous study:
Binding Energy $\Delta E = +0.514\text{eV}$

$$\Delta E = E(\text{Mu}/\alpha\text{-SiO}_2) - (E(\text{Mu})+E(\alpha\text{-SiO}_2))$$

Unstable ?

Significant lattice stress effects !?

Hydrogen isotope in α -SiO₂ Quartz

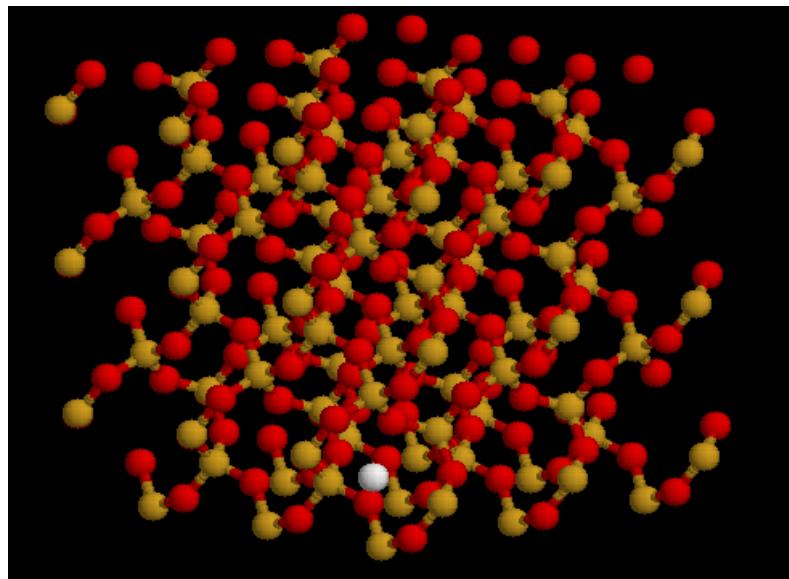
3x3x3 unit cells

Si: 81 atoms

O: 162 atoms

H: 1 atoms

1.2% hydrogen impurity

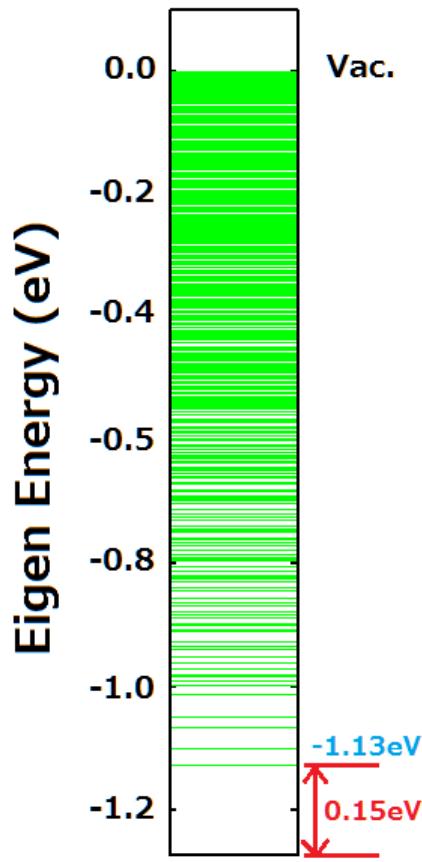


Energy: $\Delta E = -1.27\text{eV}$

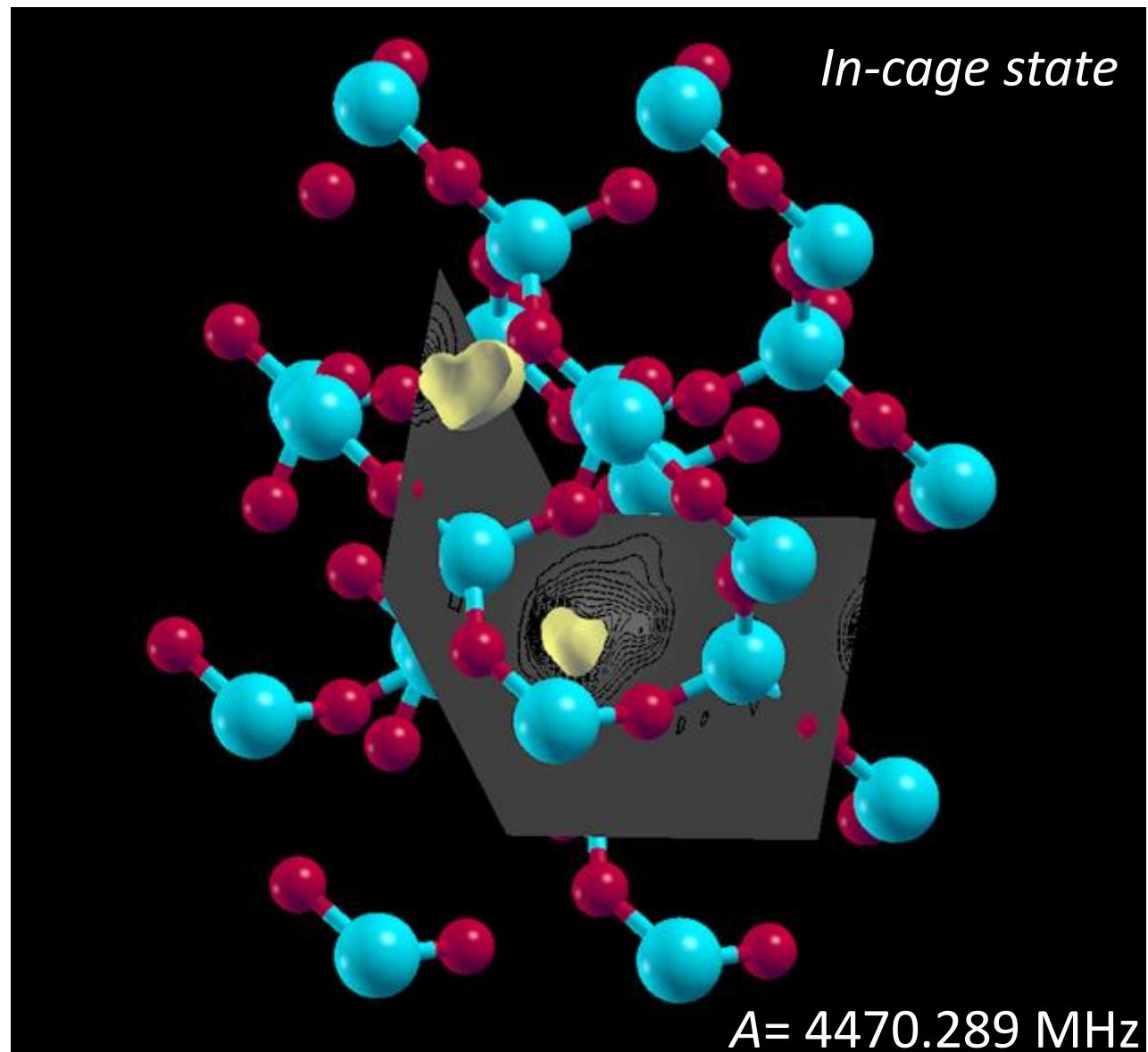
Stable !

Muonium in $\alpha\text{-SiO}_2$ Quartz

Naniwa results

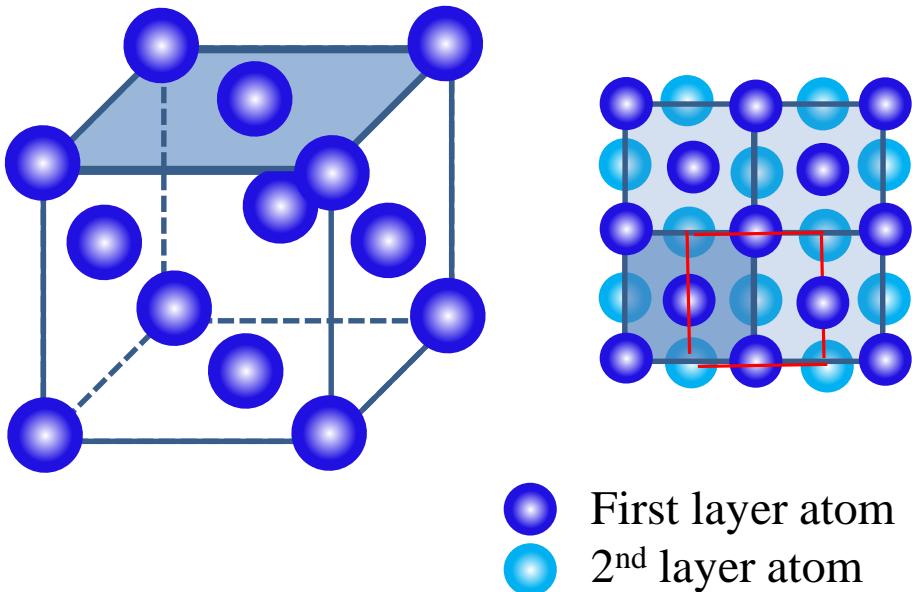


$$\Delta E = -1.13\text{ eV}$$



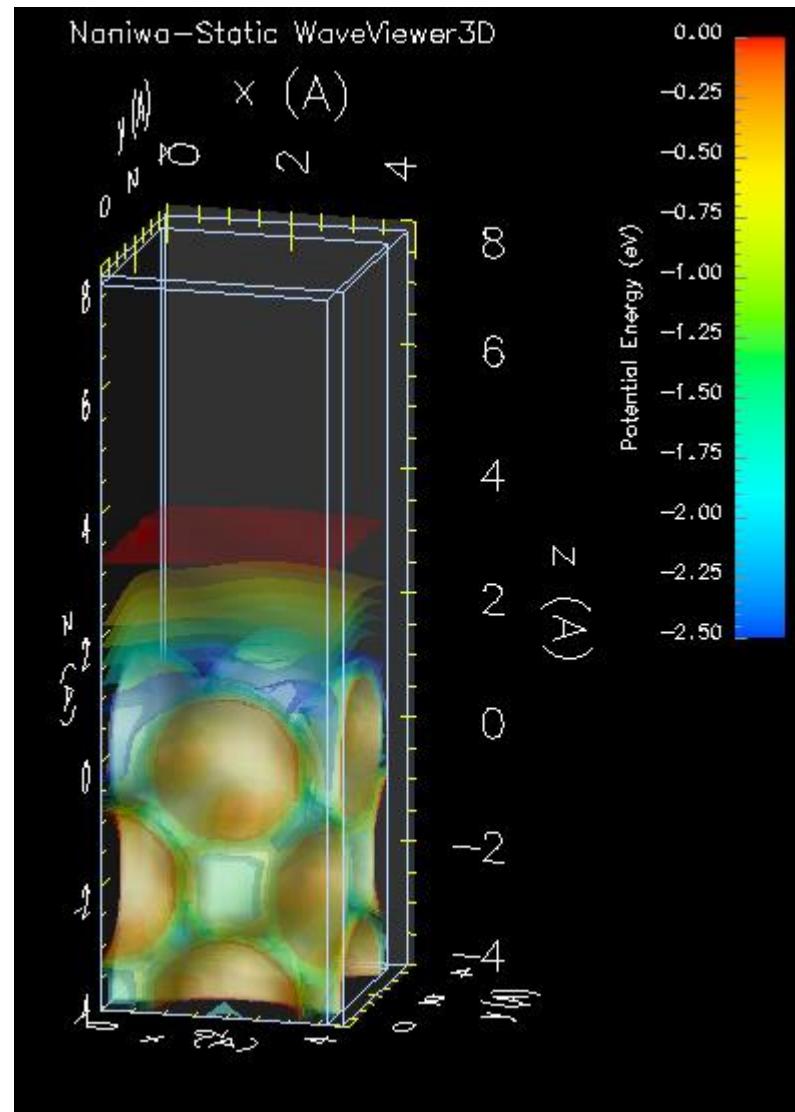
Pd (001) surfaces

(001) surface

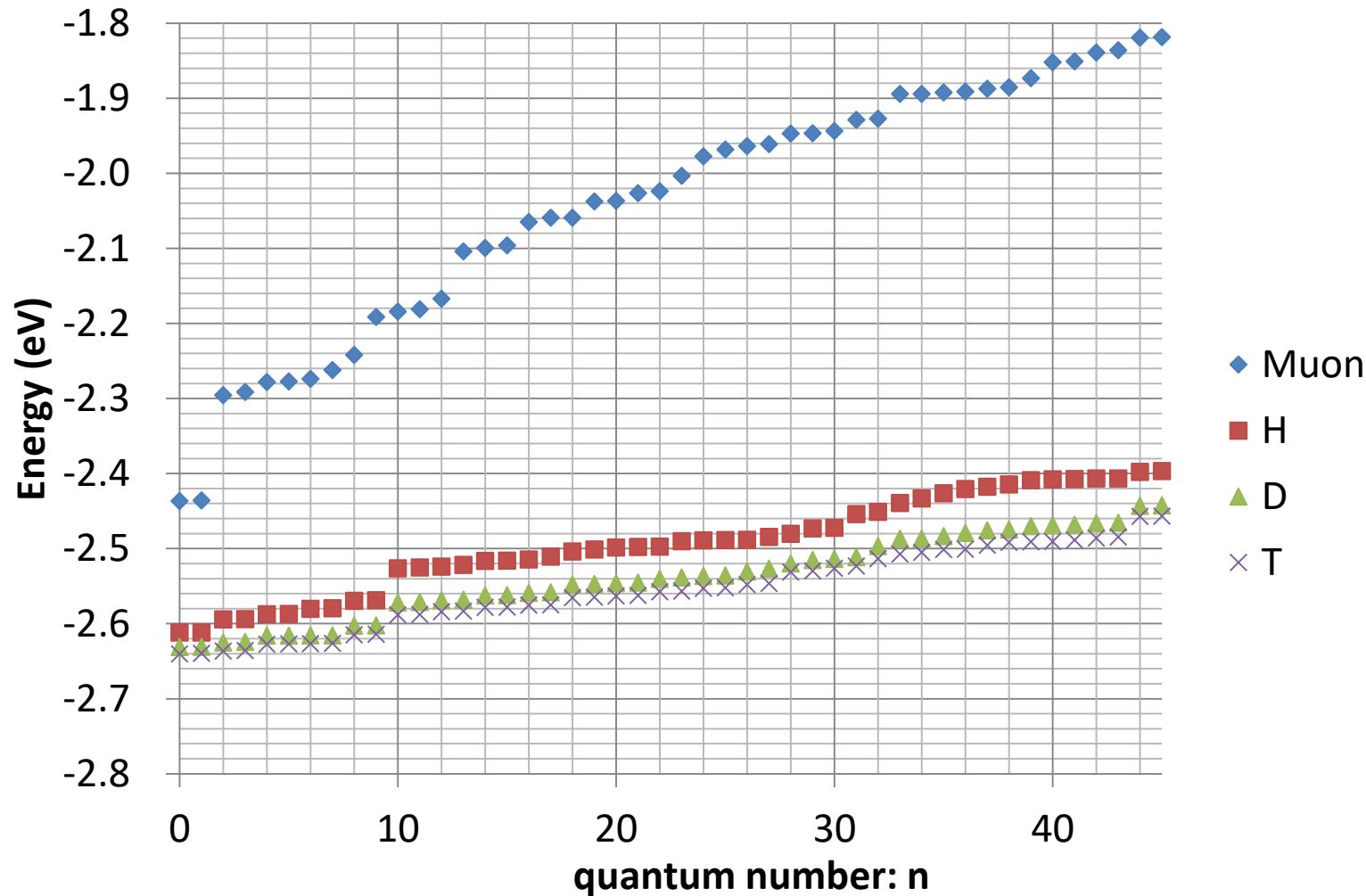


face center cubic (FCC)

lattice constant: 3.99 Å (calc.) ,
3.89 Å (exp.)

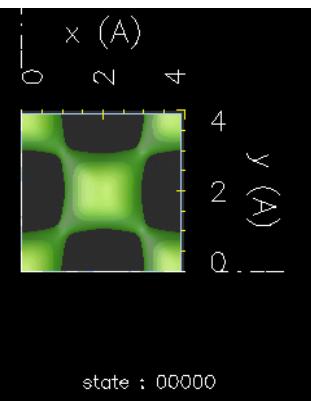


μ^+ , H, D, T on Pd (001) surfaces



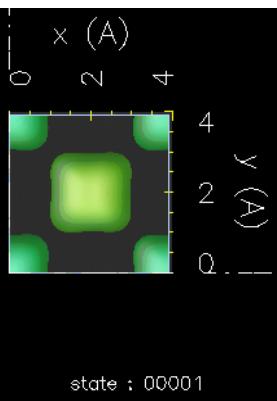
μ^+ on Pd (001) surfaces

Ground state



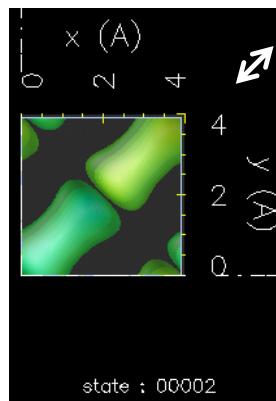
state : 00000

1st excited state



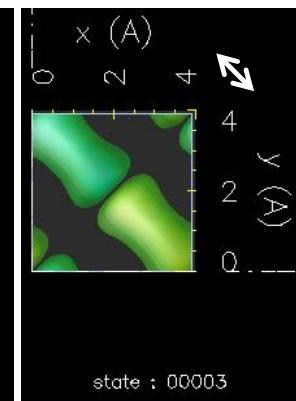
state : 00001

2nd excited state



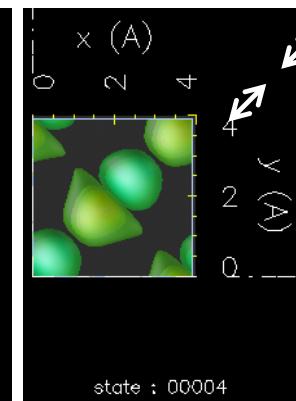
state : 00002

3rd excited state



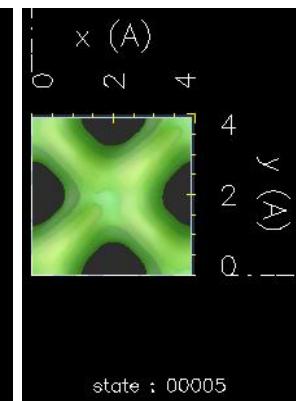
state : 00003

4th excited state

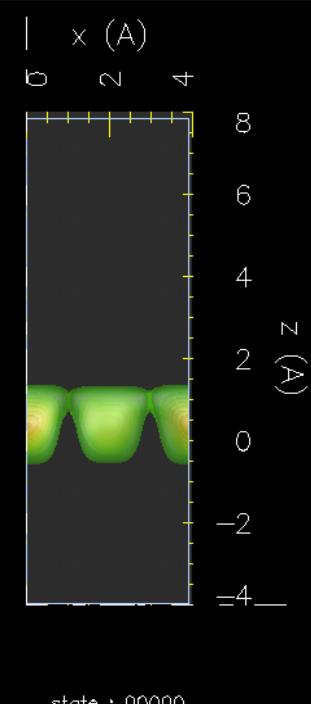


state : 00004

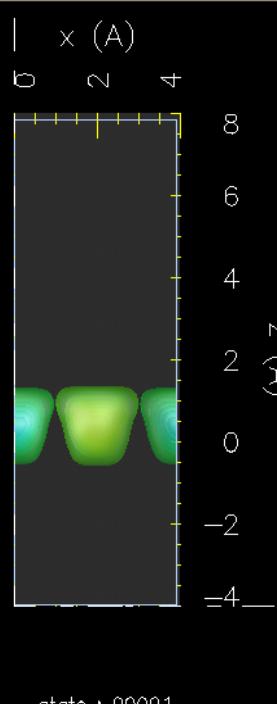
5th excited state



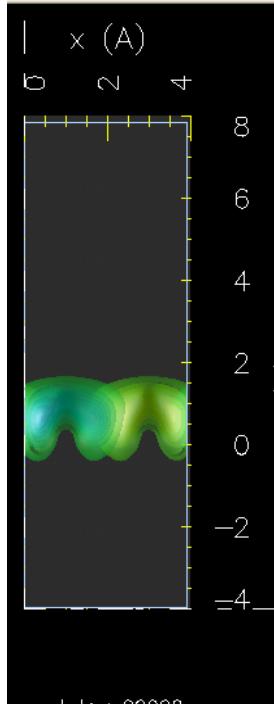
state : 00005



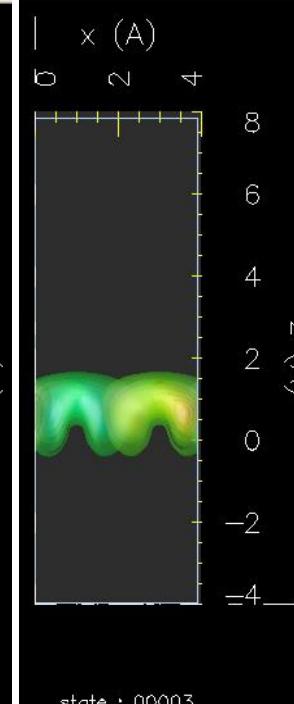
state : 00000



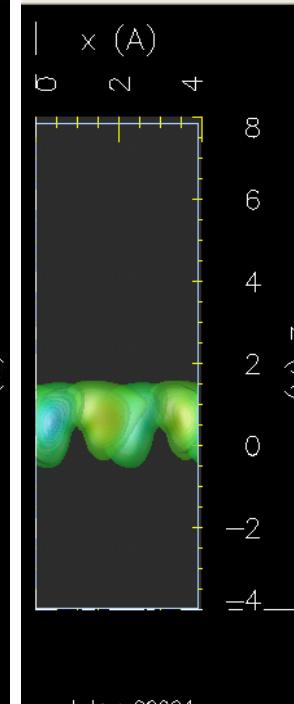
state : 00001



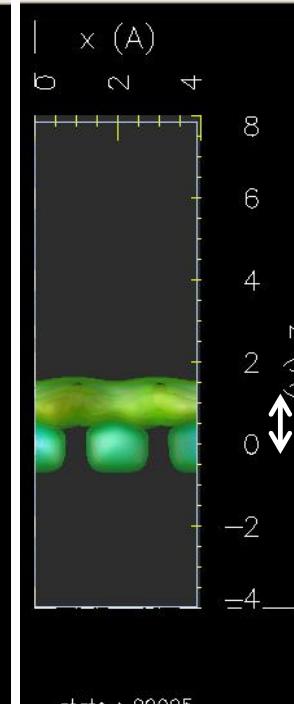
state : 00002



state : 00003



state : 00004

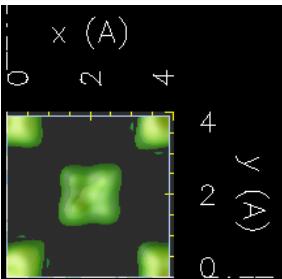


state : 00005

H on Pd (001) surfaces

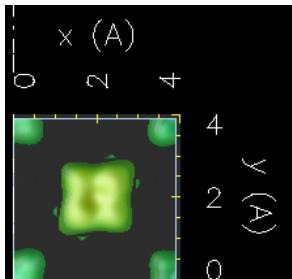


Ground state



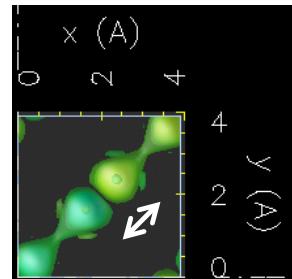
state : 00000

1st excited state



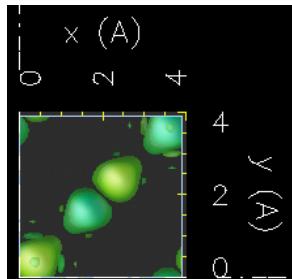
state : 00001

2nd excited state



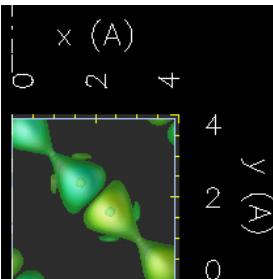
state : 00002

3rd excited state



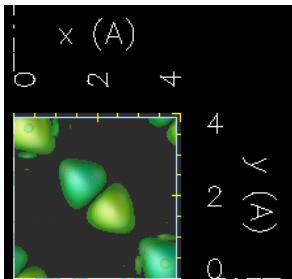
state : 00003

4th excited state

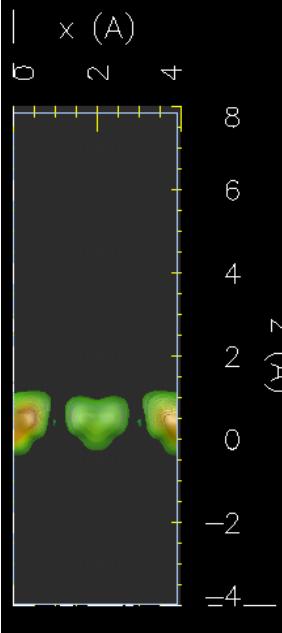


state : 00004

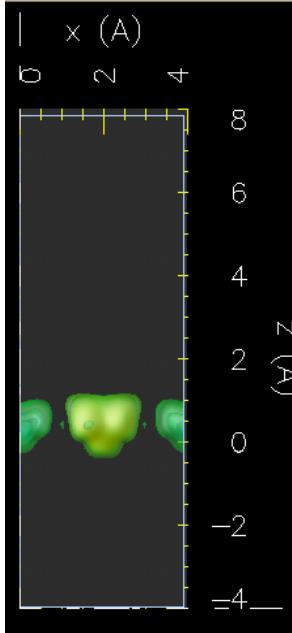
5th excited state



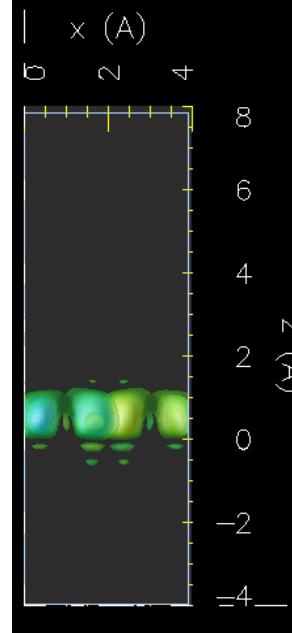
state : 00005



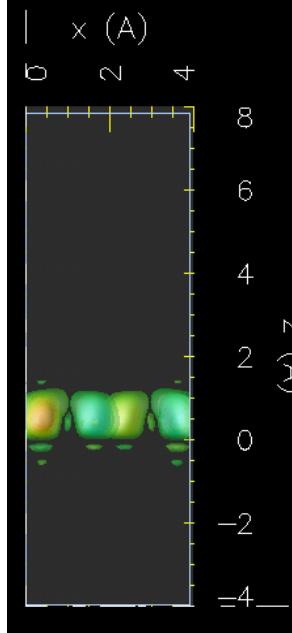
state : 00000



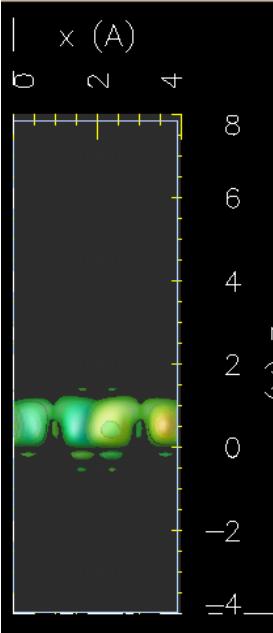
state : 00001



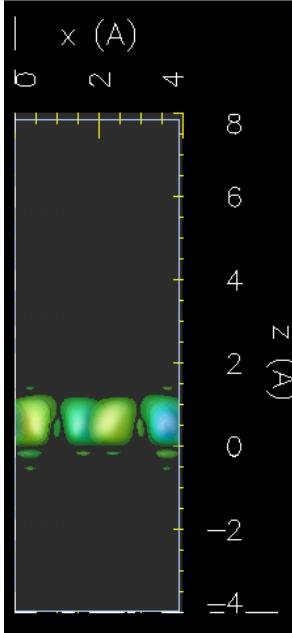
state : 00002



state : 00003



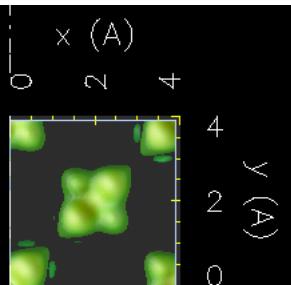
state : 00004



state : 00005

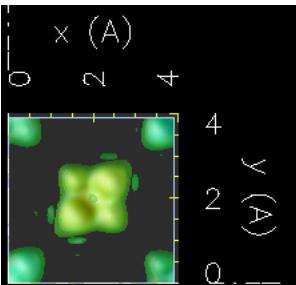
D on Pd (001) surfaces

Ground state



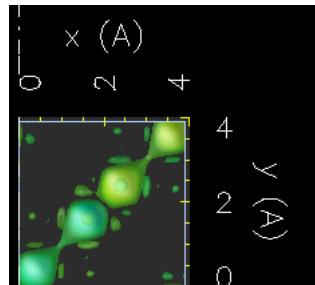
state : 00000

1st excited state



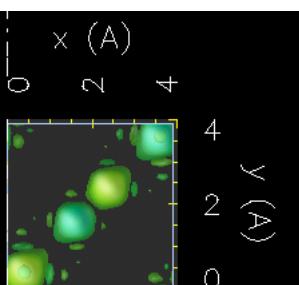
state : 00001

2nd excited state



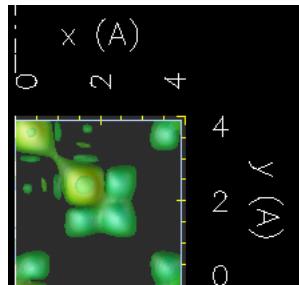
state : 00002

3rd excited state



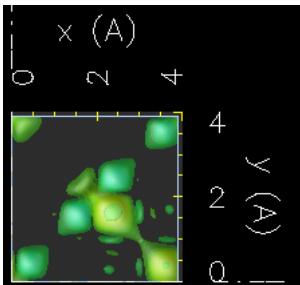
state : 00003

4th excited state



state : 00004

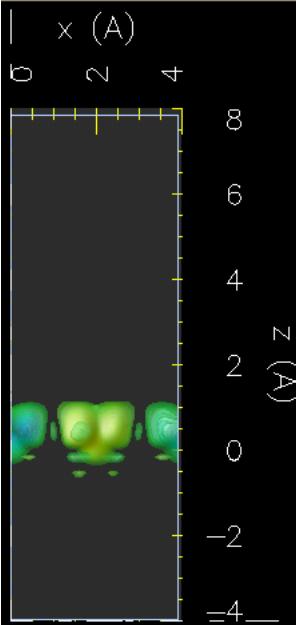
5th excited state



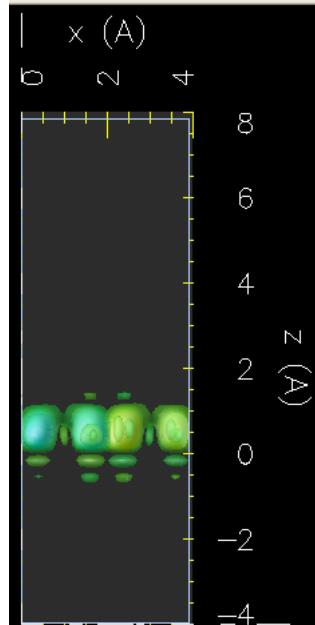
state : 00005



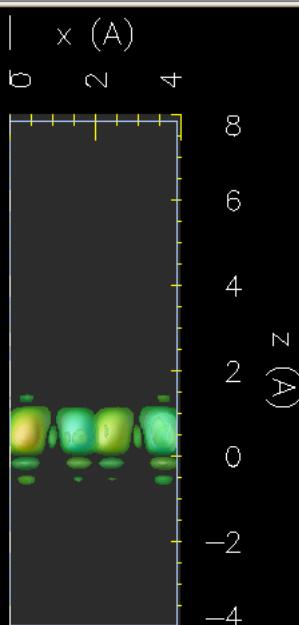
state : 00000



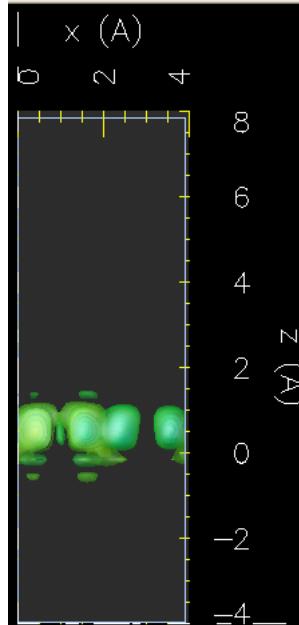
state : 00001



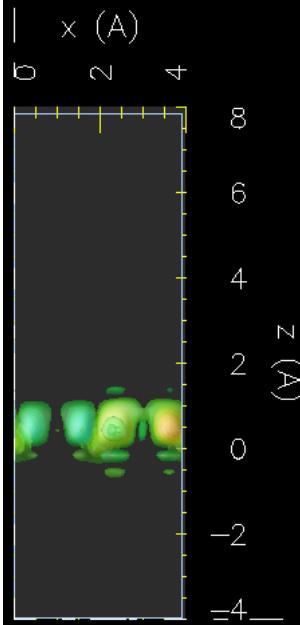
state : 00002



state : 00003



state : 00004



state : 00005

T on Pd (001) surfaces

Ground state

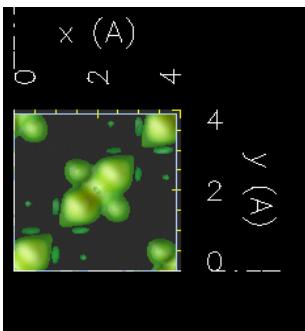


2nd excited state

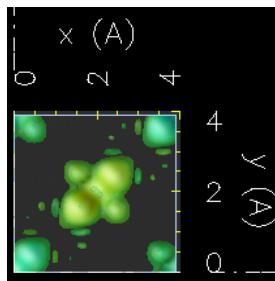
3rd excited state

4th excited state

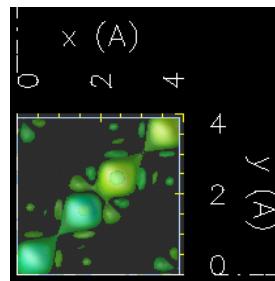
5th excited state



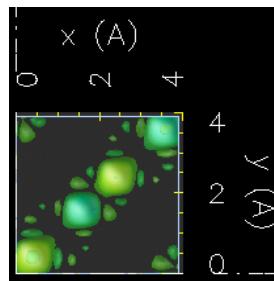
state : 00000



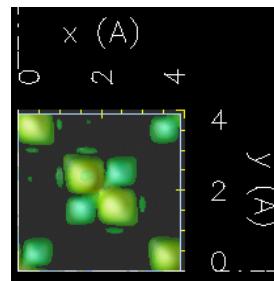
state : 00001



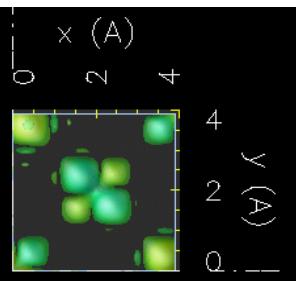
state : 00002



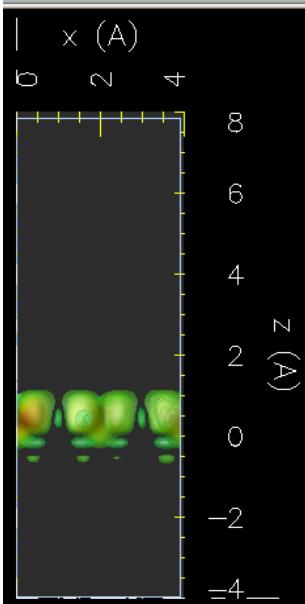
state : 00003



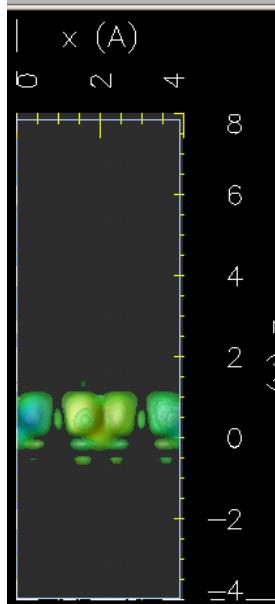
state : 0000



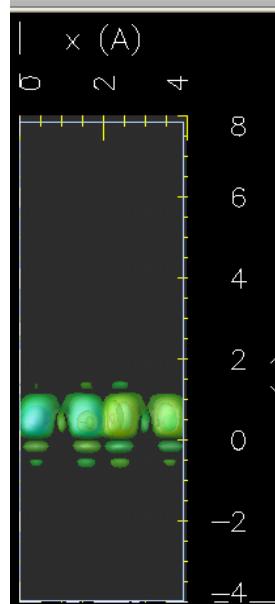
state : 00005



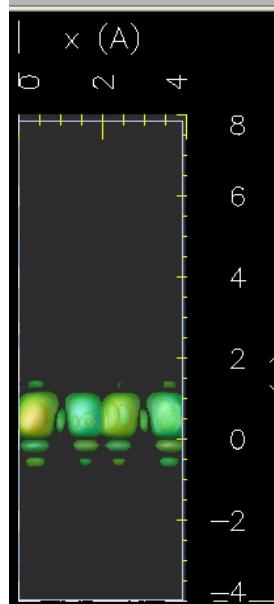
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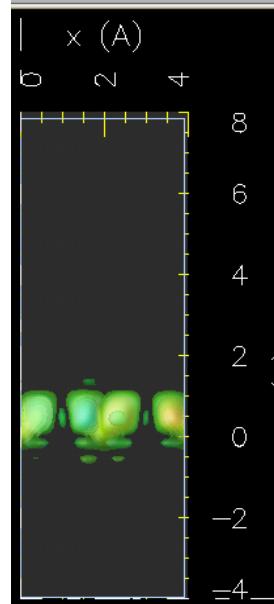
state : 0000-



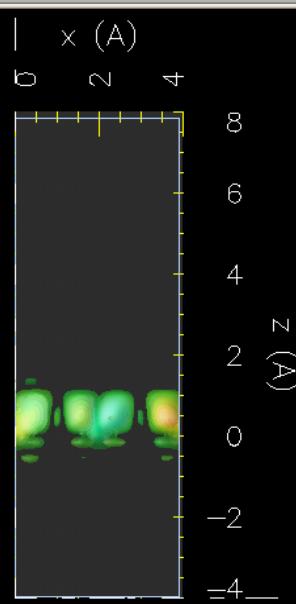
state : 00002



state : 00003

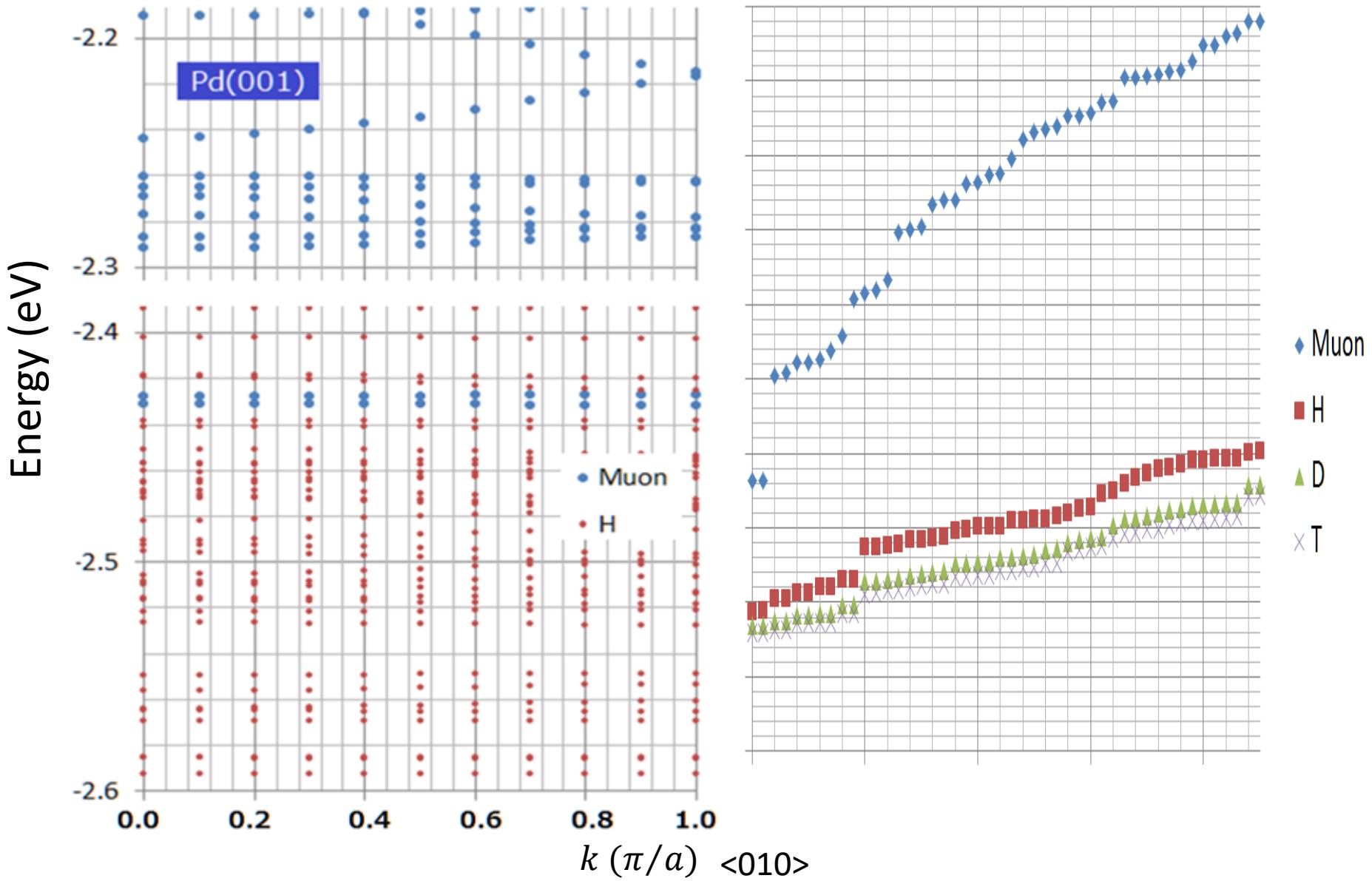


state : 0000-

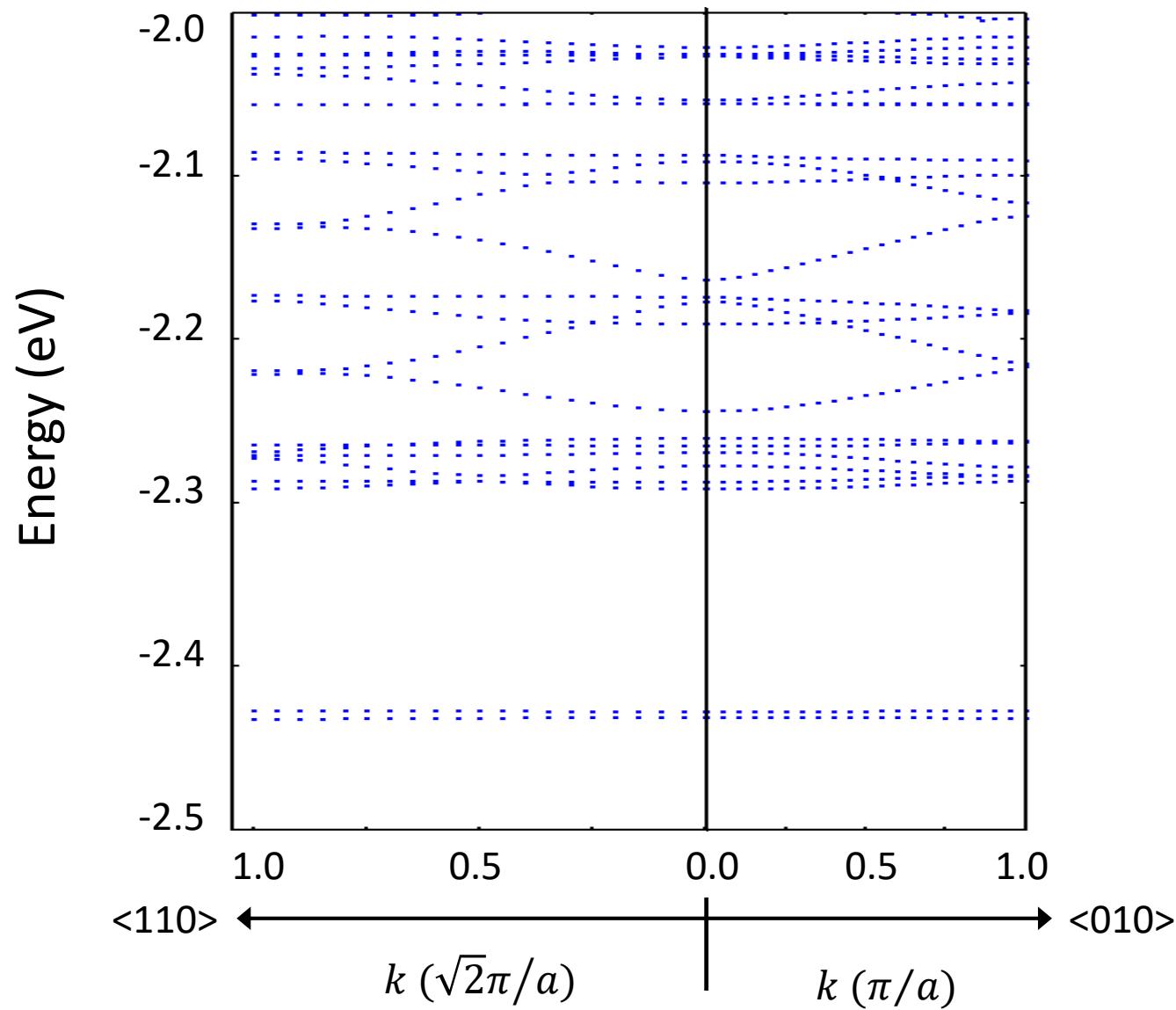


state : 00005

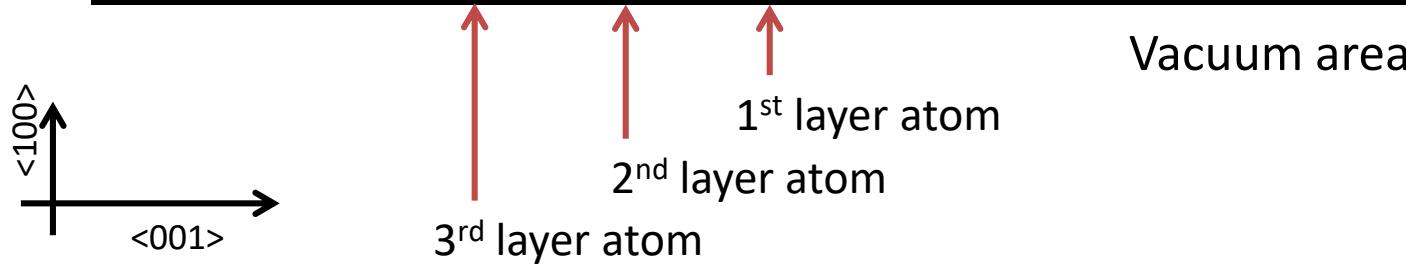
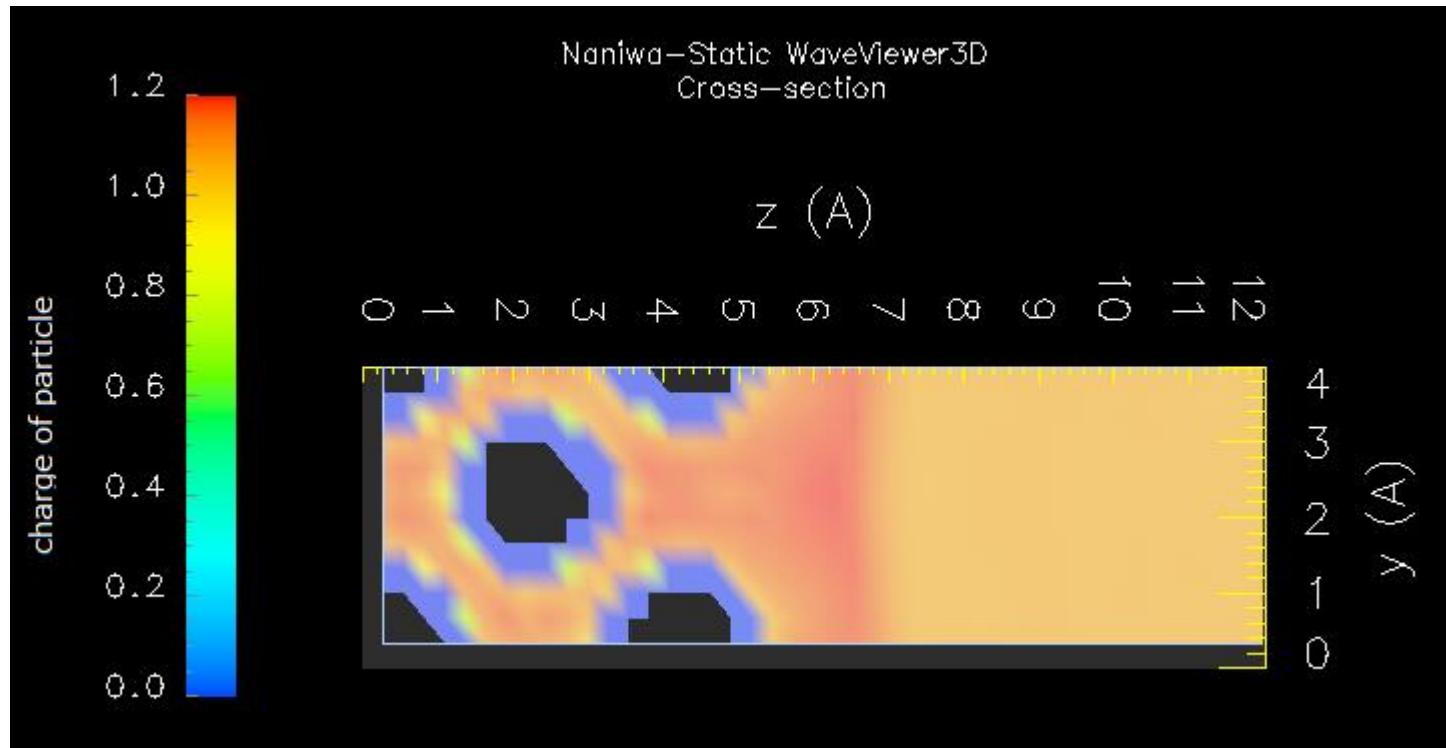
Energy Band (Wave vector dependence) of μ^+ & p^+ on Pd(001)



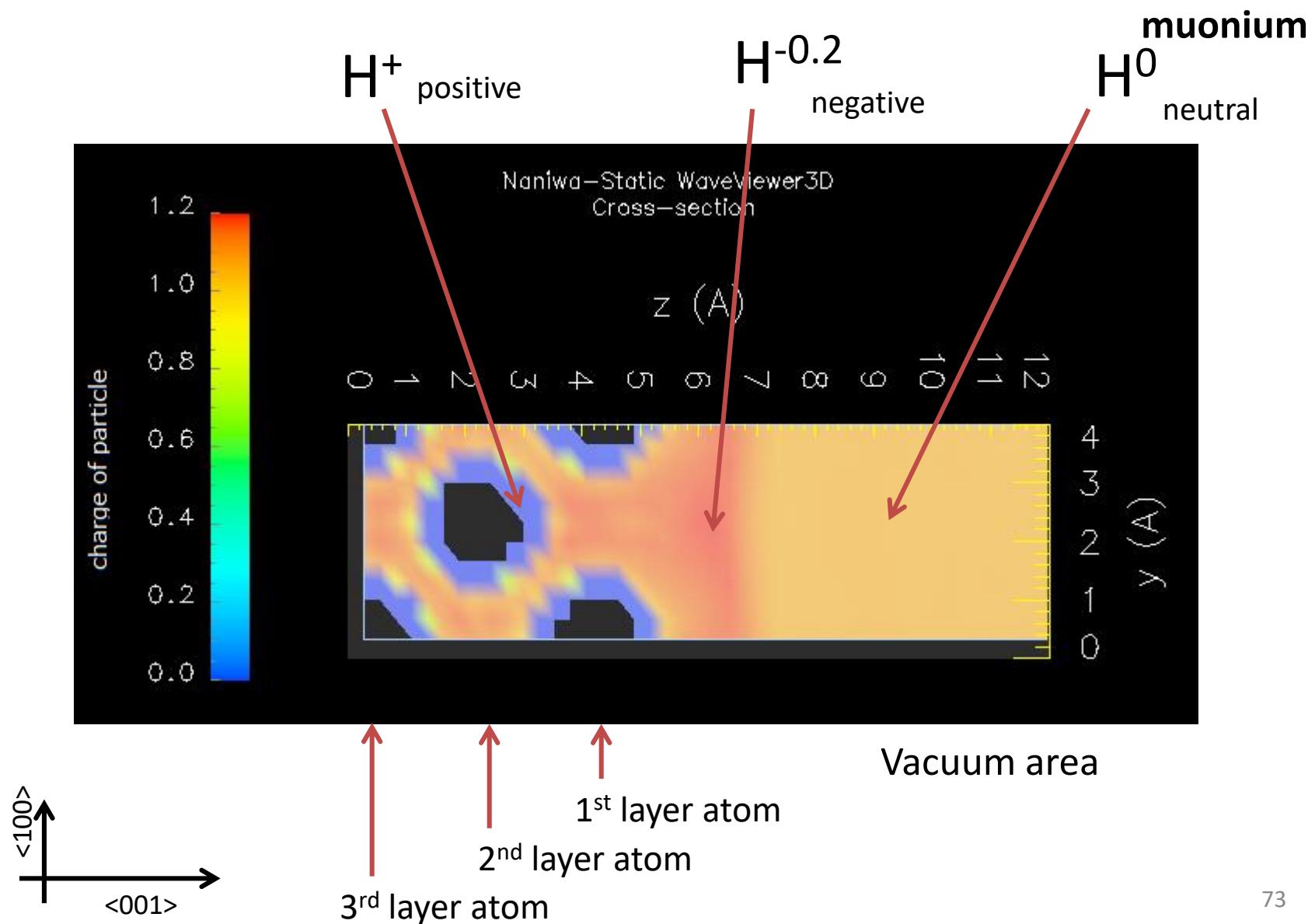
Energy Band (Wave vector dependence) of μ^+ on Pd(001)



Electron charge of target particle on Pd(001)



Charged states of target particle on Pd(001)



Naniwa codes

- We have been developing the quantum simulation code “Naniwa” for the small mass atoms on the solid surface, in the subsurface and bulk without fitting parameters.



Future Applications of Naniwa

Hydrogen related materials

- Hydrogen Fuel cell (FC) technology
- Hydrogen storage materials
- Hydrogen purification materials

Hydrogen bond ->

- Various chemical reactions in aqueous solution
--> biological material and its related reactions

Small mass atom related material

- Li : lithium-ion secondary battery
- Oxygen: cathode reaction of FC