

# ecalj CMD2019 course menu

Sep.4<sup>th</sup> Wed 4 hours, Thus 6 hours , Fri 2 hours.

- Introduction  
What we can do in ecalj?
- ecalj package  
Install  
Minimum usage  
Band structure calculation. GW calculation.
- Theory  
Band theory. LMTO+LAPW method, GW method
- Usage, Read output and Practice

Give a glance to <https://github.com/tkotani/ecalj/>

(This documents contains some theory --- I will change this documents more...)

# The **PMT** method: a new linearized method

$$\text{PMT} = \text{APW} + \text{MTO}$$

Muffin-tin

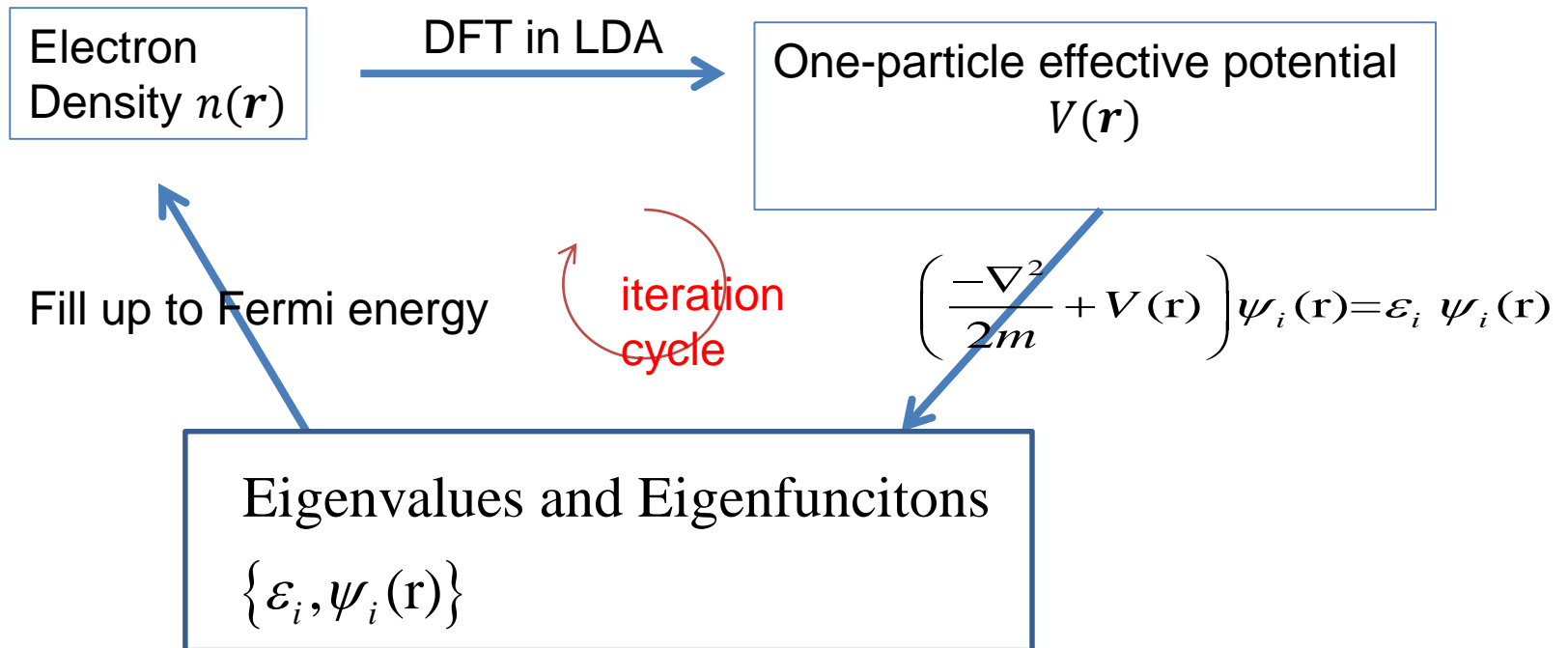
*Takao Kotani (tottori-u)*

Key point:

1. Iteration cycle.
2. eigenfunctions are expanded with APWs(augmented plane wave) and MTOs(muffin-tin orbital)

# Independent particle picture and total energy

These can be obtained by the density functional theory (DFT) in LDA.



This iteration cycle until converged = total energy minimization

How to represent density and so on in computer?  
How to solve it numerically?

## **Lists of the Full-potential methods**

- **KKR**

- **Pseudopotential method**

- **PAW**

- **LMTO**

- **LAPW**



Linearized xxx methods

(why do we call them "linearized" method?  
→ from the view of "exact" APW method)

# Finite basis set

## •Basis set (finite number of basis)

→ We assume eigenfunctions are given as:

$$\psi_p(\mathbf{r}) = \sum_j \alpha_p^j F_j(\mathbf{r}) \quad \{F_j\} : j = 1, 2, \dots, N$$

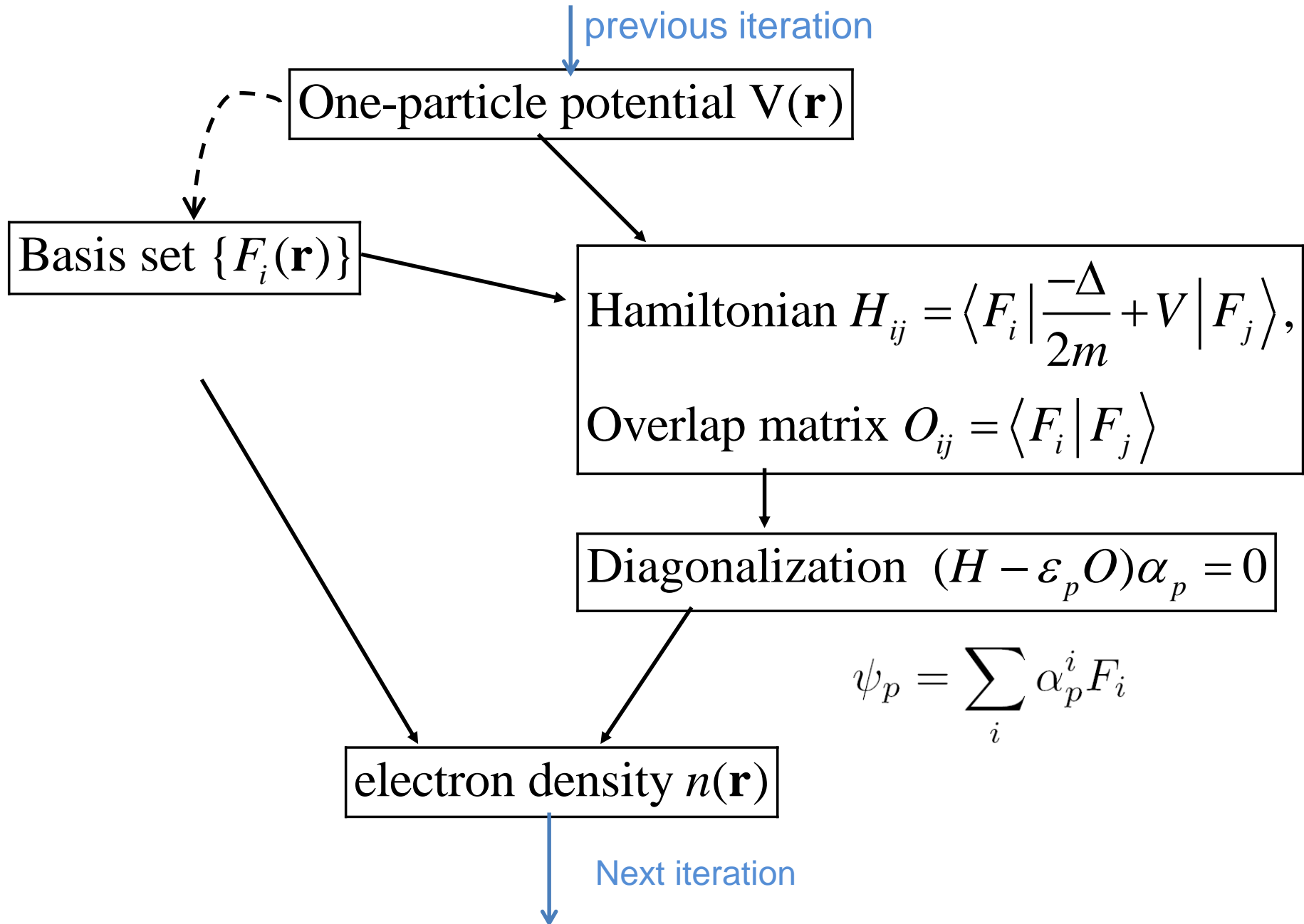


$$\text{Hamiltonian } H_{ij} = \langle F_i | \frac{-\Delta}{2m} + V | F_j \rangle,$$
$$\text{Overlap matrix } O_{ij} = \langle F_i | F_j \rangle$$

$$\left( H_{ij} - \varepsilon O_{ij} \right) \alpha^j = 0$$

Finite dimension problem(as the same as LCAO/Gaussian)

## iteration cycle



## How to choose the good basis?

- APW (augmented plane wave)
- MTO (muffin-tin orbital)

are the names of the basis functions.

Both of them are made by “augmentation”.

### My conclusion:

To overcome shortcomings in APW basis and MTO basis, we should use both of APW and MTO together. →this means the PMT method

But wait...

What is the APW and MTO? → next page.

1 Lecture 11: Atomic Sphere Methods: Augmentation, Green's Functions and Linearization Electronic Structure of Condensed Matter, Physics 598SCM

2 Muffin Tin Division of Space

3 Muffin Tin Approximation

4 Augmentation

5 Augmented Plane Waves - Slater

6 Augmented Plane Waves II

7 Augmented Plane Waves III

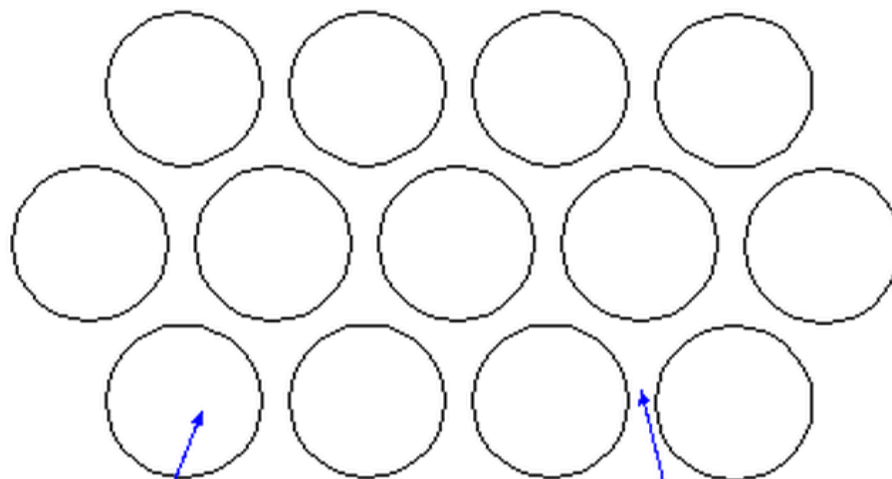
8 APW - Bands of Cu

9 APW - Bands of Transition metals

10 The KKR (multiple

# Muffin Tin Division of Space

A definition – not an approximation



Sphere around each atom

MT regions

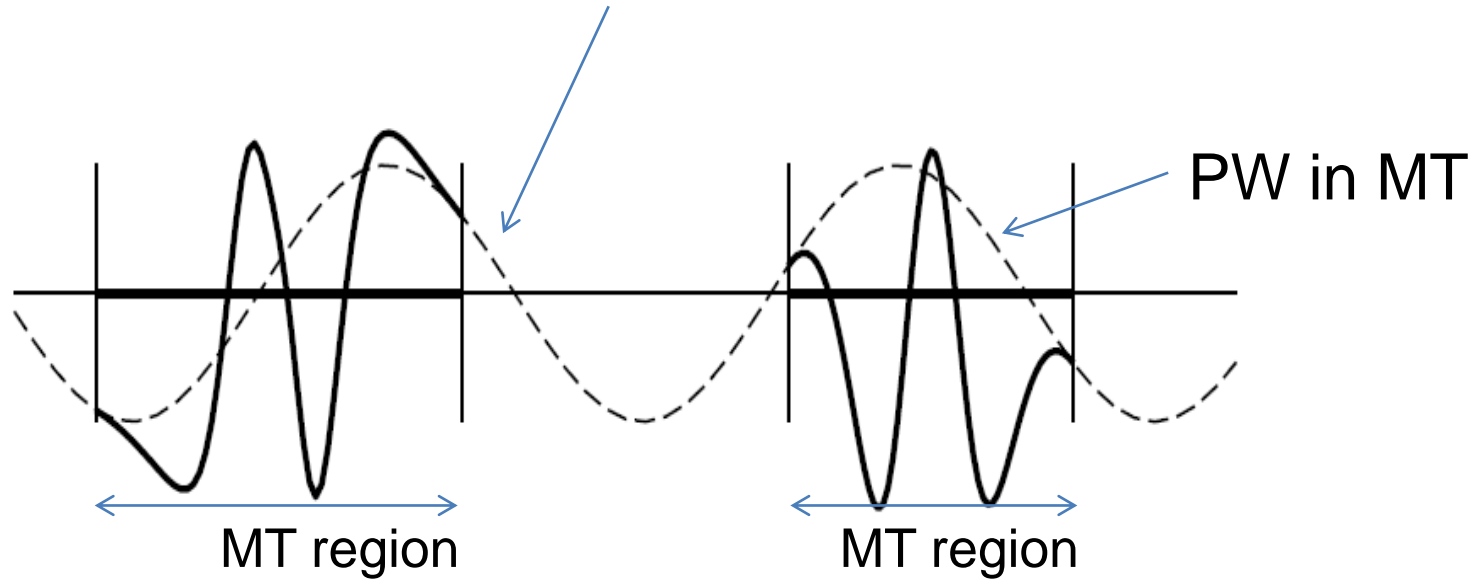
Interstitial regions between atoms

Interstitial region



Envelope function = PlaneWave(PW)

APW

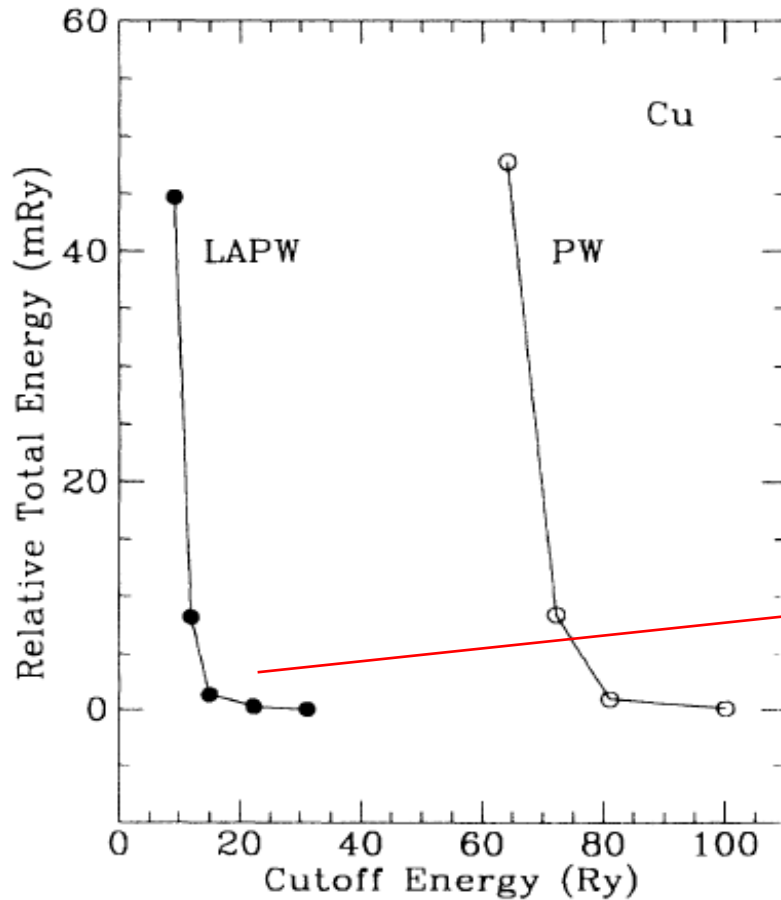


### 3-components

PW + Atomic-like part — Counter part:  
0<sup>th</sup> 1<sup>st</sup> PW within MT 2<sup>nd</sup>

- Good for Na(3s), high energy bands.
- Not so good for Cu(3d), O(2p)
- Systematic

# Linearized APW



fcc Cu

plane wave is not so efficient for 3d

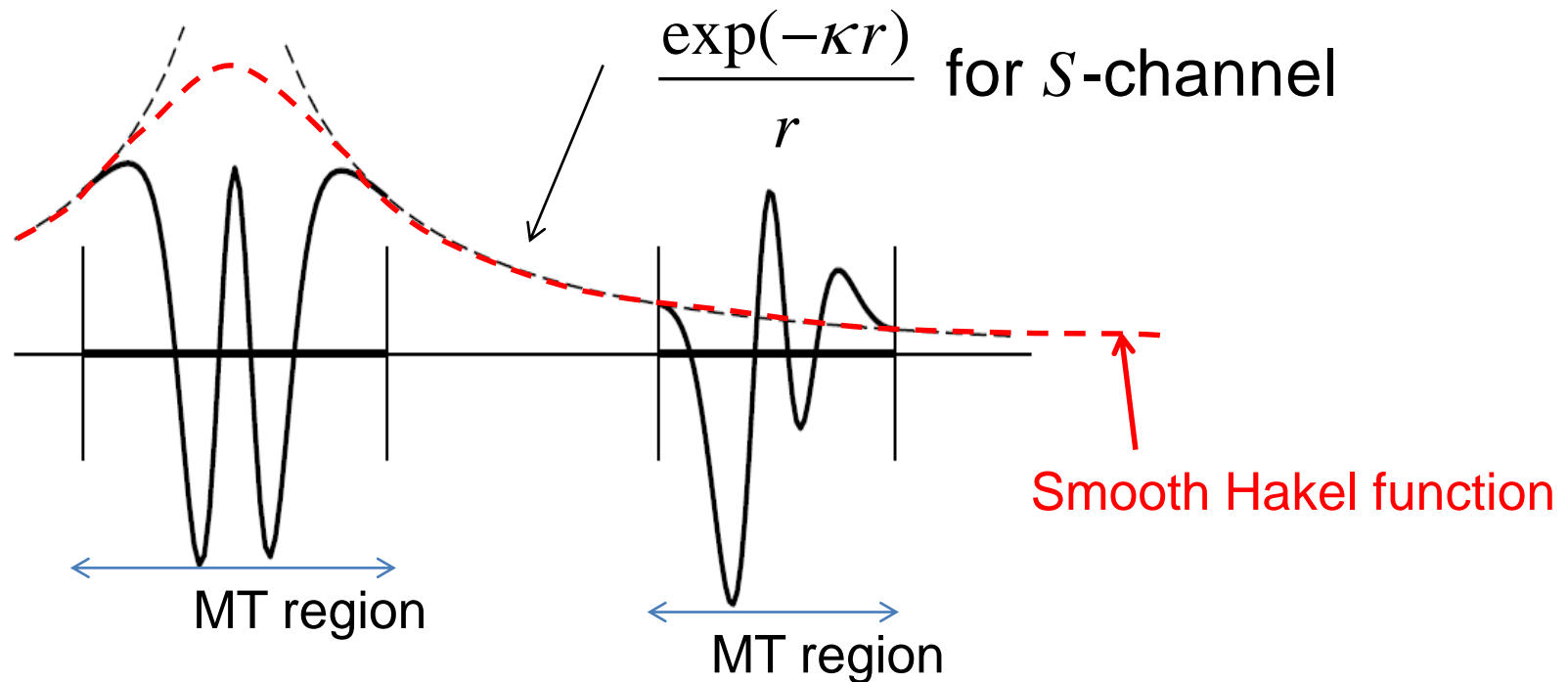
15Ry or more for ~1mRy convergence

FIG. 1. Relative total energy of fcc Cu plotted against plane-wave cutoff energy.

D.Singh et al PRB49,17424

# MTO

Atom-centered Hankel function, e.g,



3-components

Hankel  $0^{\text{th}}$  + Atomic-like part  $1^{\text{st}}$  - counter part of Hankel  $2^{\text{nd}}$

- Good for localized orbitals such as Cu(3d), O(2p)
- Not so good for extended states, surface.
- Not systematic

$$\text{PMT} = \text{A}\underline{\text{PW}} + \underline{\text{MTO}}$$

***T.K and M.van Schilfgaard***

Phys. Rev. B 81, 125117 (2010)

***T.K, H. Kino, and H.Akai***

Supercell calculations from H<sub>2</sub> through Kr<sub>2</sub>.

Almost automatic setting of MTOs with APWs

(Energy cutoff 3~4Ry) .

J. Phys. Soc. Jpn. 82, 124714, (2013)

J. Phys. Soc. Jpn. 84, 034702 (2015)

# Basis function

MT center at  $\mathbf{R}$ . Radius  $R$ .

a basis  $F_j(\mathbf{r})$  consists of

$$F_{0j}(\mathbf{r}),$$

Envelope function  
PW or smHankel

$$F_{1j}(\mathbf{r}),$$

Atomic-like parts  
for  $|\mathbf{r}| < R$

$$F_{2j}(\mathbf{r})$$

Counter part  
for  $|\mathbf{r}| < R$

**Augmentation parts**

**Cutoff:  $l \leq l_{\max} \sim 4$ , Radial-part expansion**

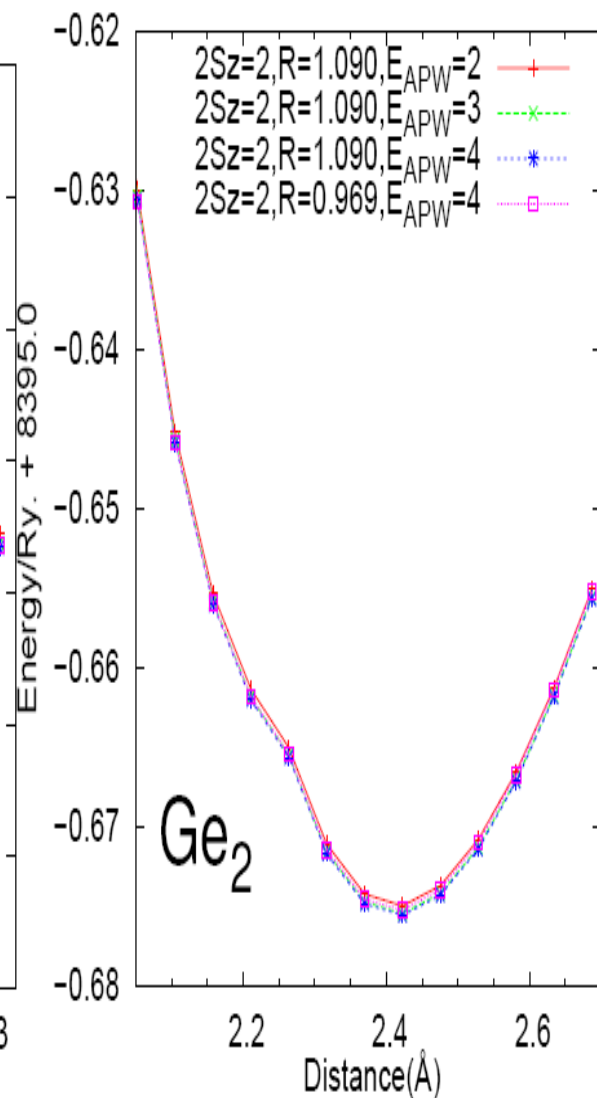
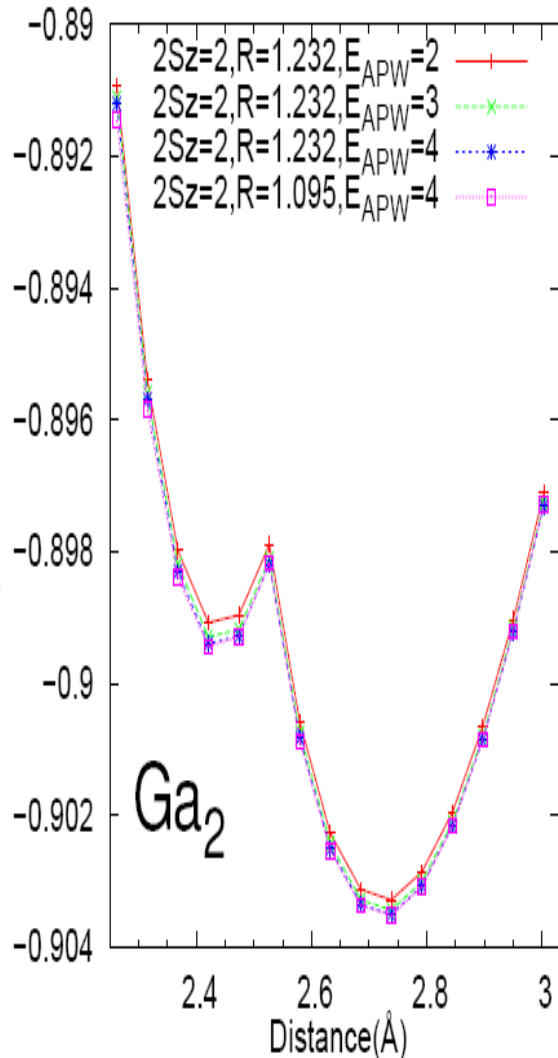
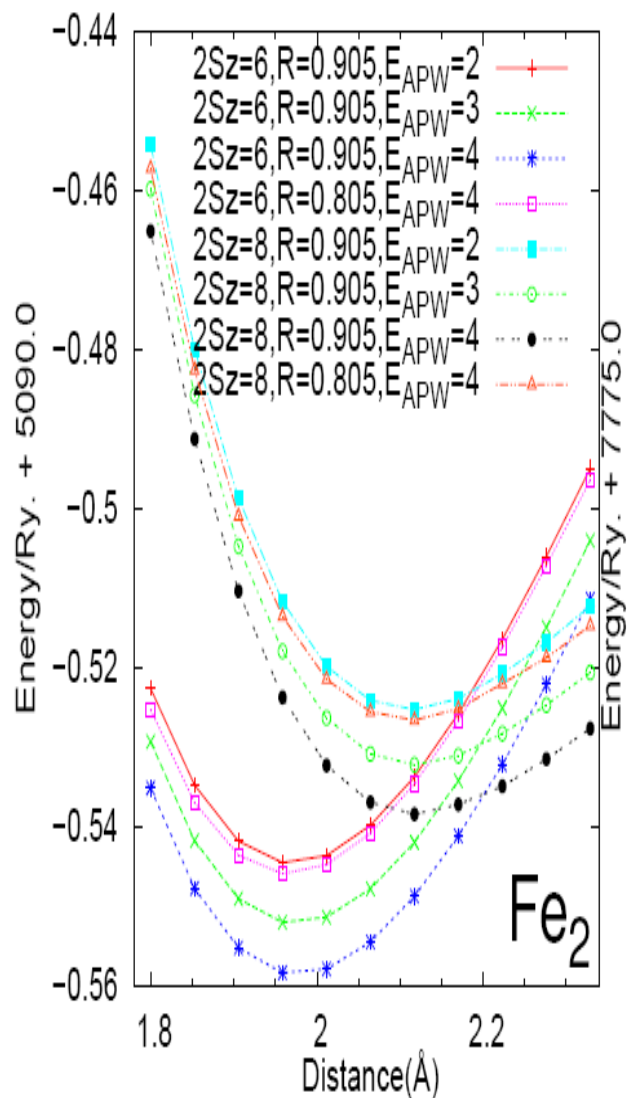
$$F_j(\mathbf{r}) = F_{0j}(\mathbf{r}) + F_{1j}(\mathbf{r} - \mathbf{R}) - F_{2j}(\mathbf{r} - \mathbf{R})$$

Electron density  $n(\mathbf{r})$  and potential  $V(\mathbf{r})$  are expanded in a similar manner.

**Local orbitals(lo):** basis functions which are Non-zero only within MTs.

e.g, see <http://www.wien2k.at/lapw/index.html>

This is also used together.



$N_{basis} = 1081 + 36 \times 2, \quad 1973 + 36 \times 2, \quad 3025 + 36 \times 2$

## Comparison with Gaussian

		$r_e$ (Å)	$D_e$ (Kcal/mol)	$\omega_e$ (cm <sup>-1</sup> )
$H_2, 2S_z=0$	PMT	0.749	104.678	4317.959
	PMT(NR)	0.750	104.764	4311.202
	GTO	0.752	104.552	4311.816
$O_2, 2S_z=2$	PMT	1.218	143.741	1564.787
	PMT(NR)	1.218	144.984	1568.867
	GTO	1.220	<del>139.815</del>	1554.249
	VASP		143.3	
$Cr_2, 2S_z=0$	PMT	1.591	32.833	813.296
	PMT(NR)	1.589	30.191	818.483
	GTO	1.595	26.192	808.148
$Fe_2, 2S_z=6$	PMT	1.977	57.596	397.673
	PMT(NR)	1.991	58.770	386.597
	GTO	2.012	56.902	397.228
$Cu_2, 2S_z=0$	PMT	2.218	51.169	269.326
	PMT(NR)	2.251	48.503	254.321
	GTO	2.251	48.645	255.768

NR: non relativistic      GTO: 6-311+G(d,p)



# Quasiparticle self-consistent GW

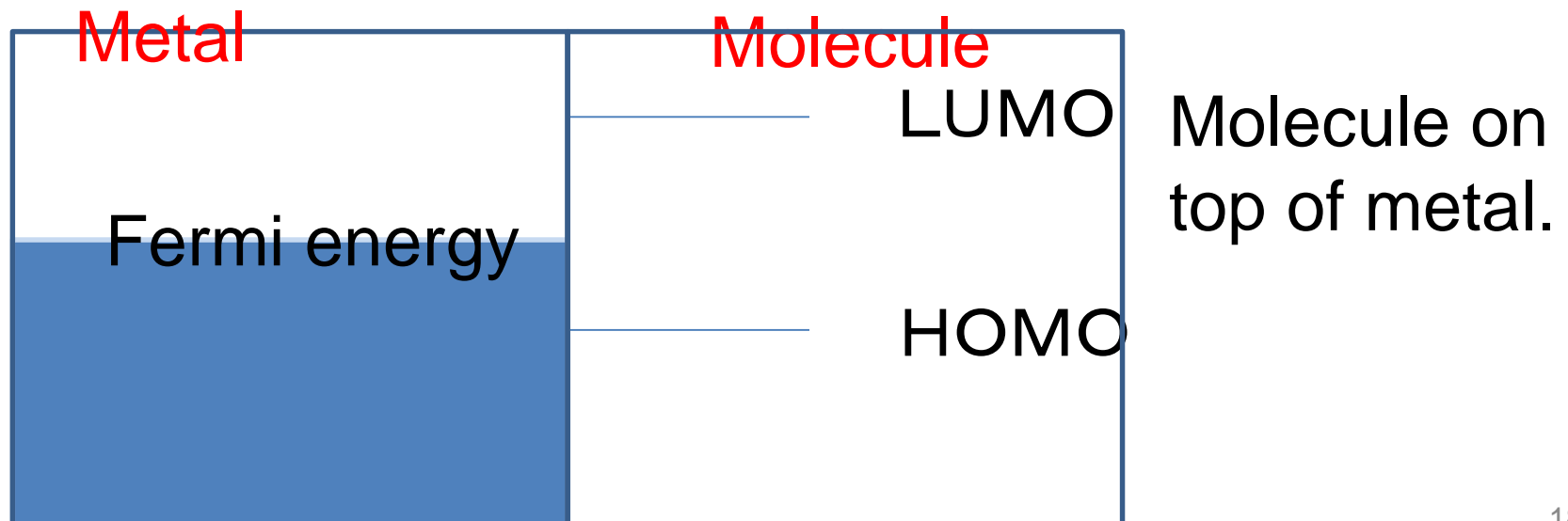
Takao kotani(*tottori university*)

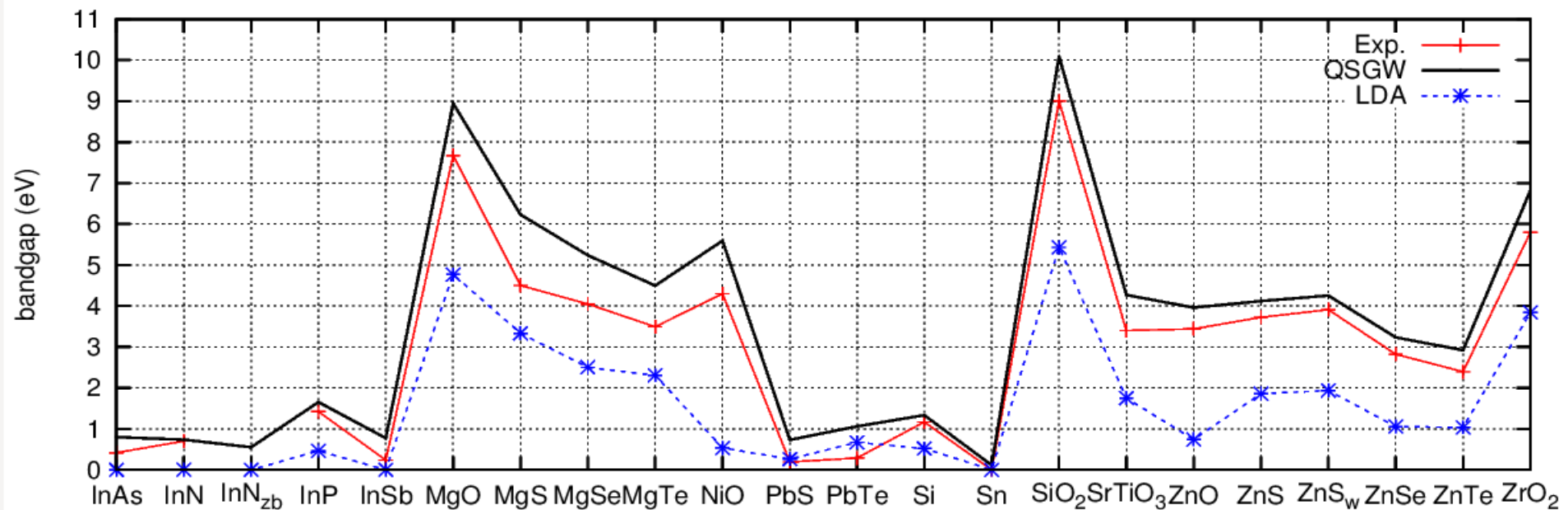
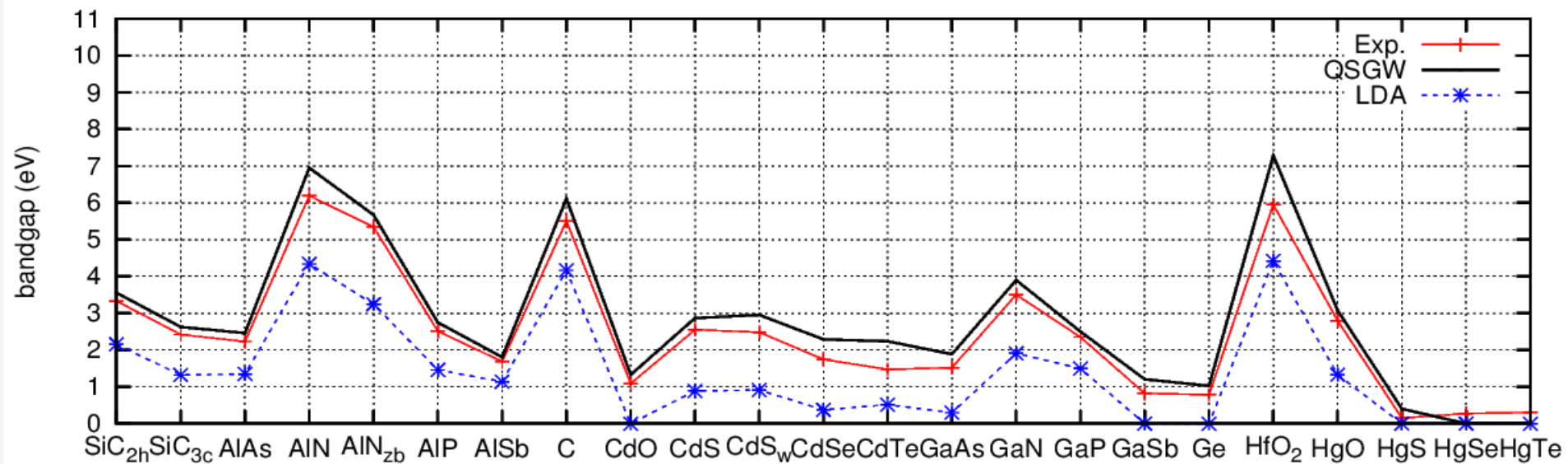
- Mean field theory.  
What the eigenvalue means?
- GW, and QSGW method  
Minimum and previous results.

# Why QSGW ?

We need good independent particle picture in order to calculate linear responses (magnetic optical, transport...).  $H_0$  (eigenvalue and eigenfunctions) are required.

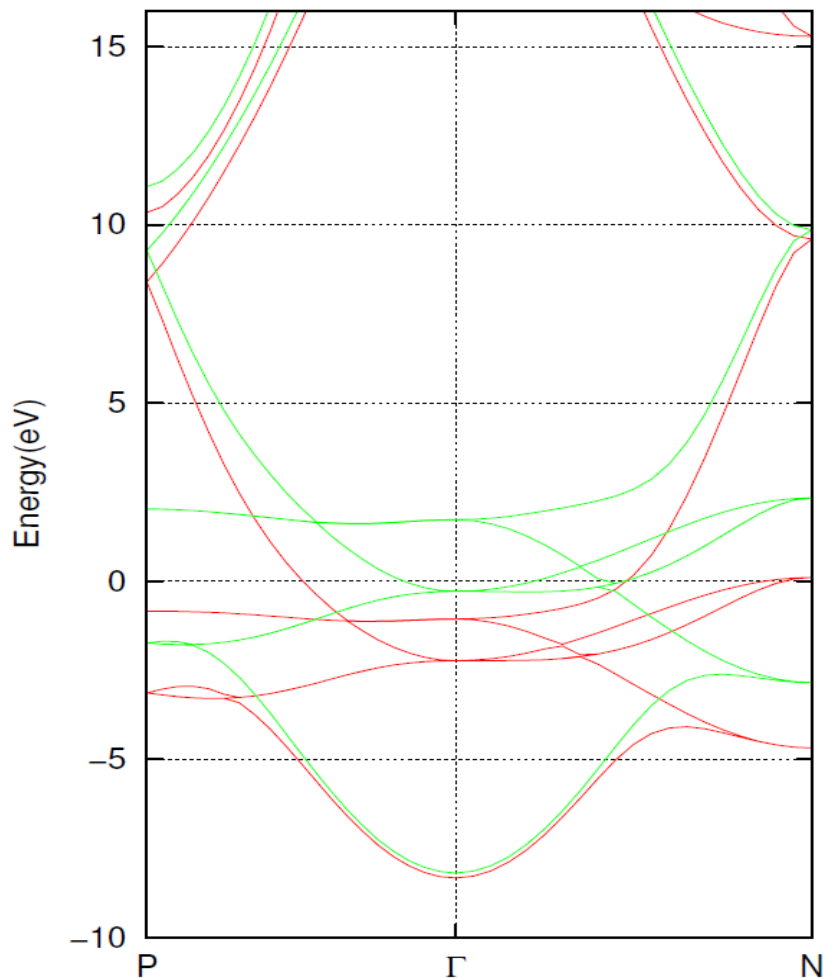
- \* **Band gap, Effective mass**
- \* **Relative position of levels.**  
LUMO—HOMO, Fermi energy...



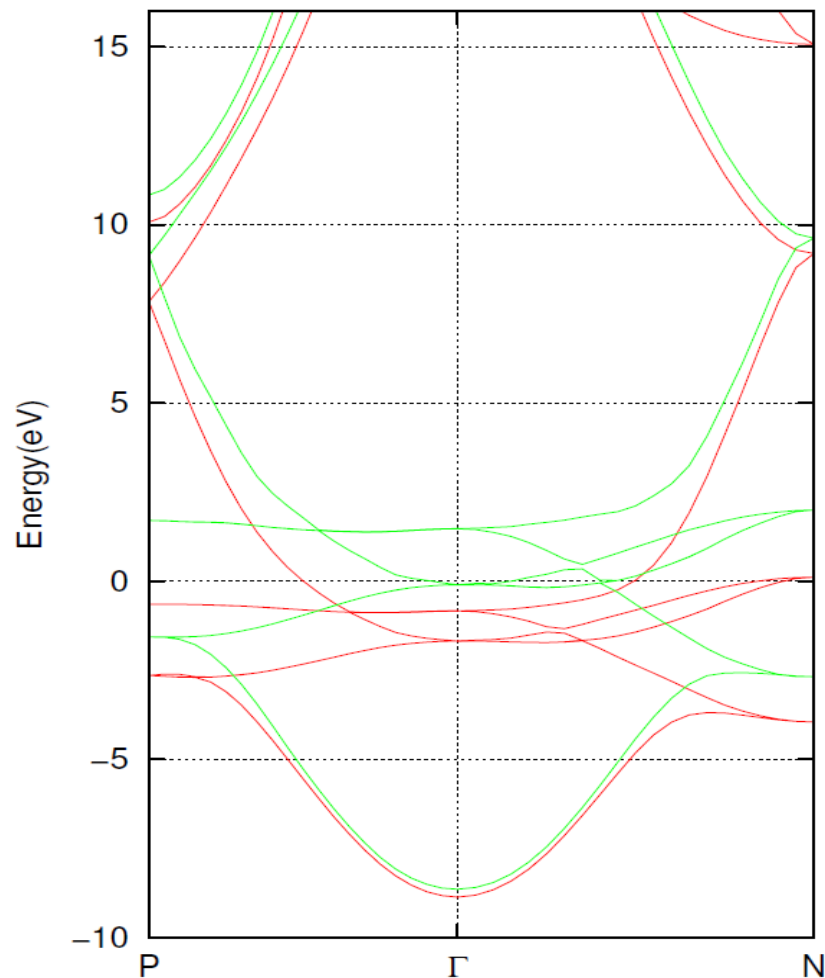


Experimental Band gap (eV)

GGA: bcc Fe



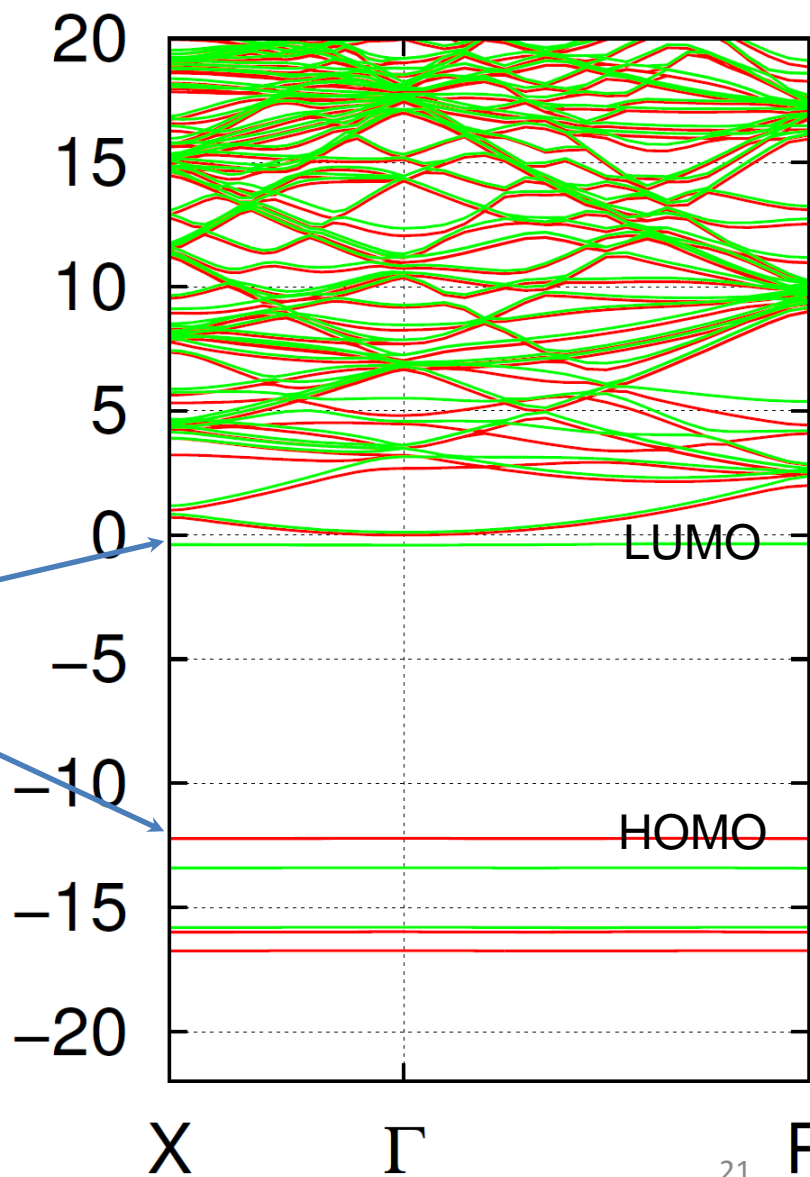
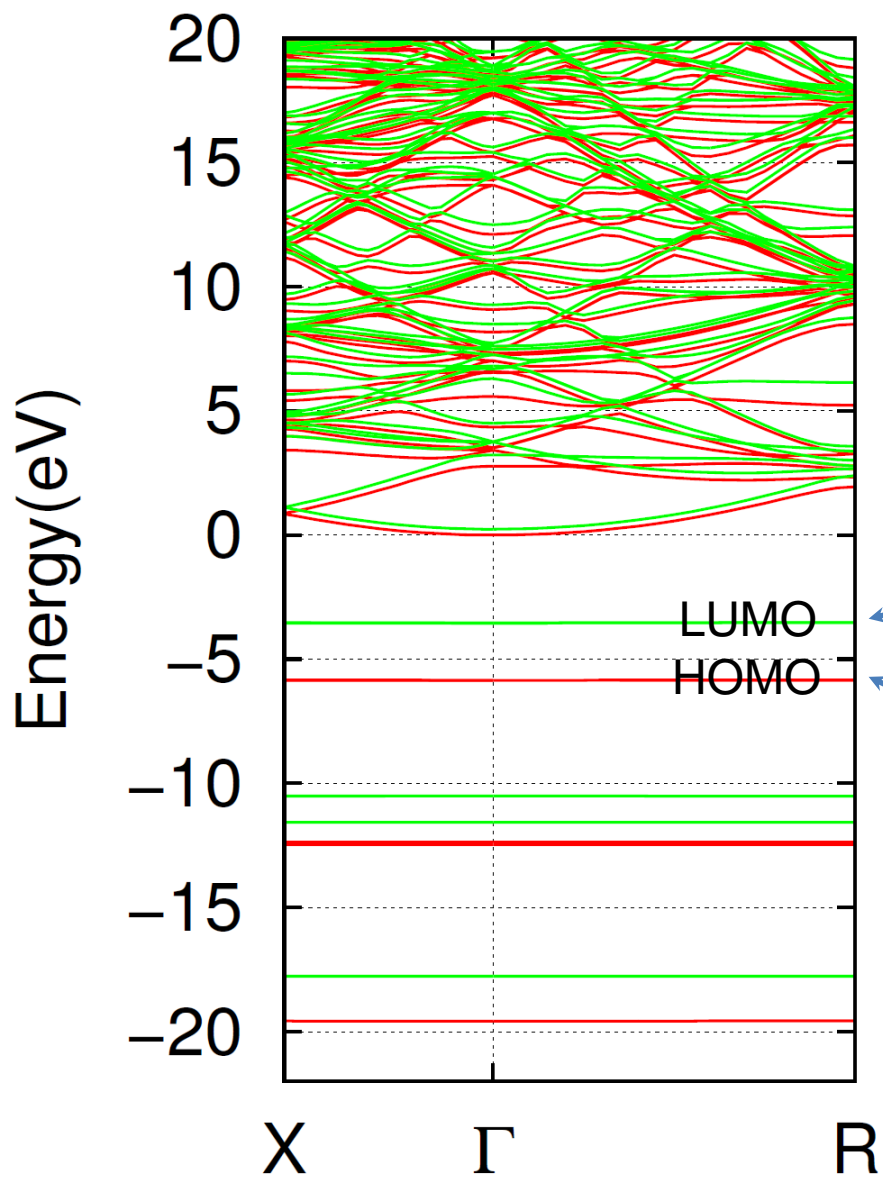
QSGW: bcc Fe



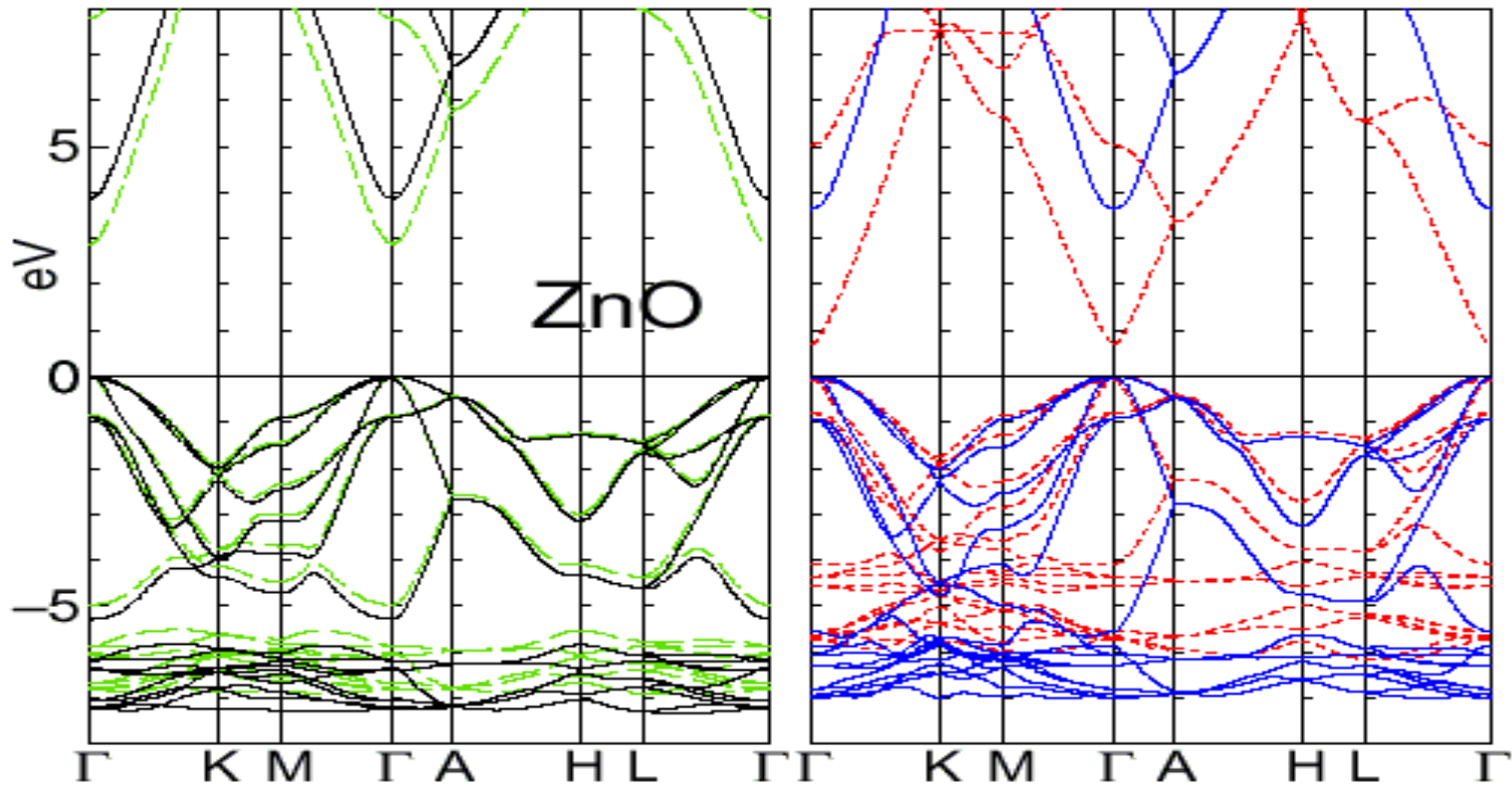
金属的なもの→QSGWでOK.  
 ハイブリッド法では場合によりうまくいかない。

GGA: O<sub>2</sub> in supercell (12a.u.)<sup>3</sup>cell

QSGW



計算の収束性は不明...



Black:QSGW

3.87eV

Red:LDA

0.71eV

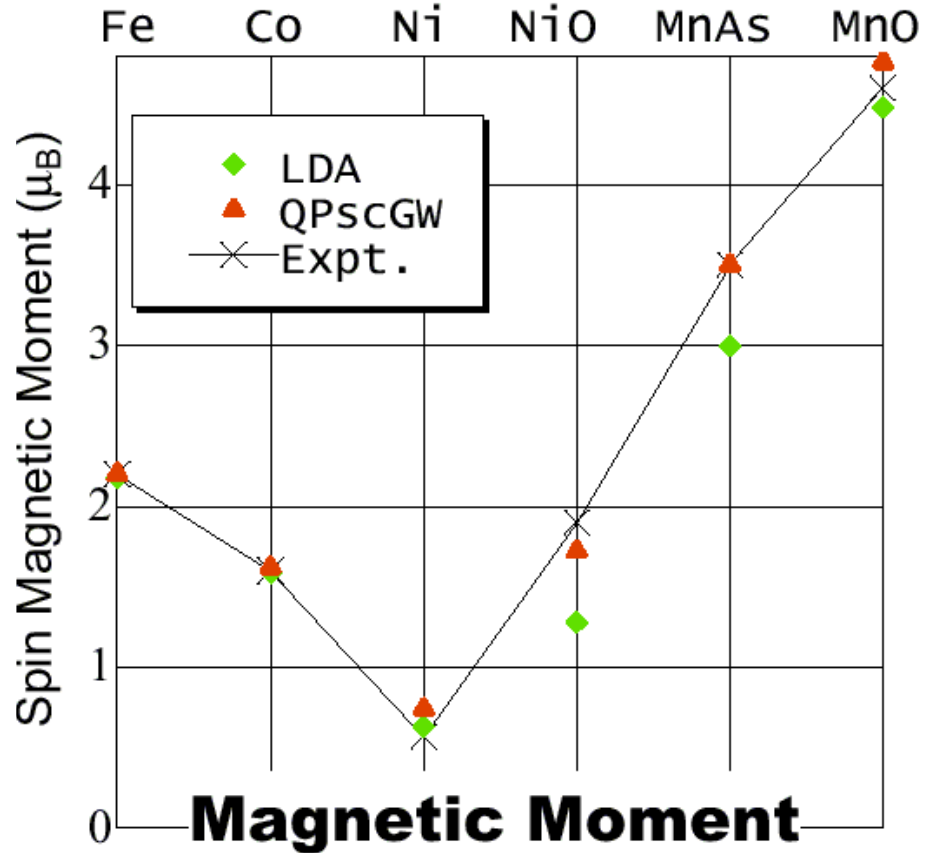
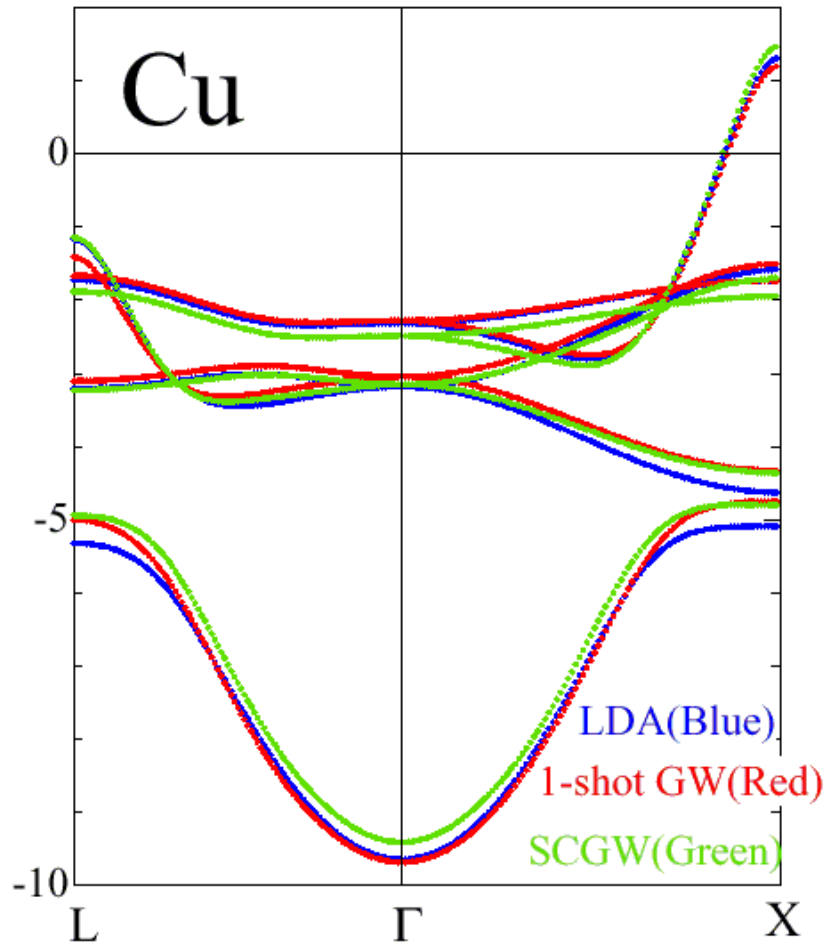
Green: $G^{LDA}W^{LDA}$  (Z=1, Offdiagonal included) 3.00eV

Blue:e-only self-consistency 3.64eV

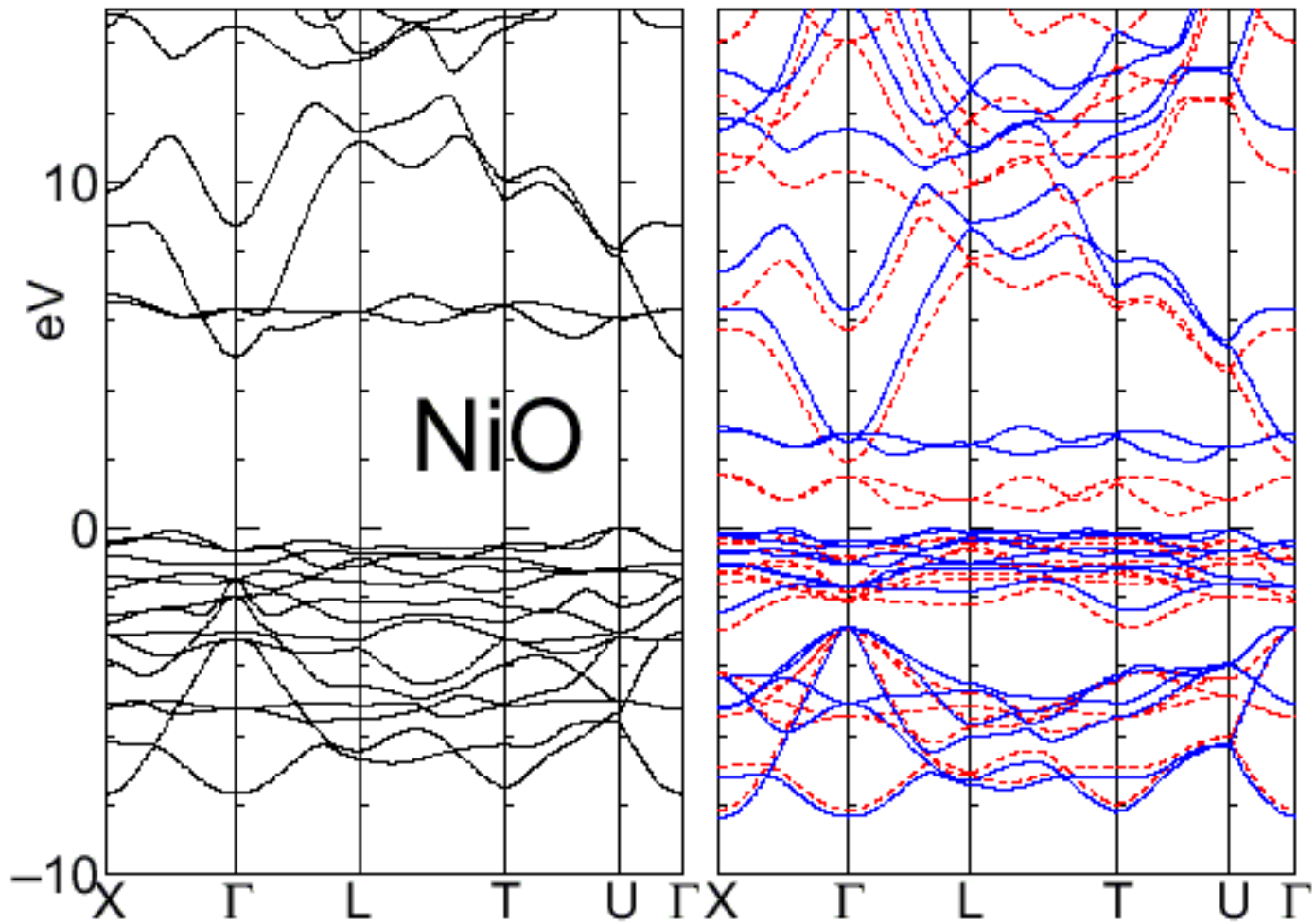
**Experiment(+correction) 3.60eV**

Kotani et al PRB76,165106(2007)

# d systems



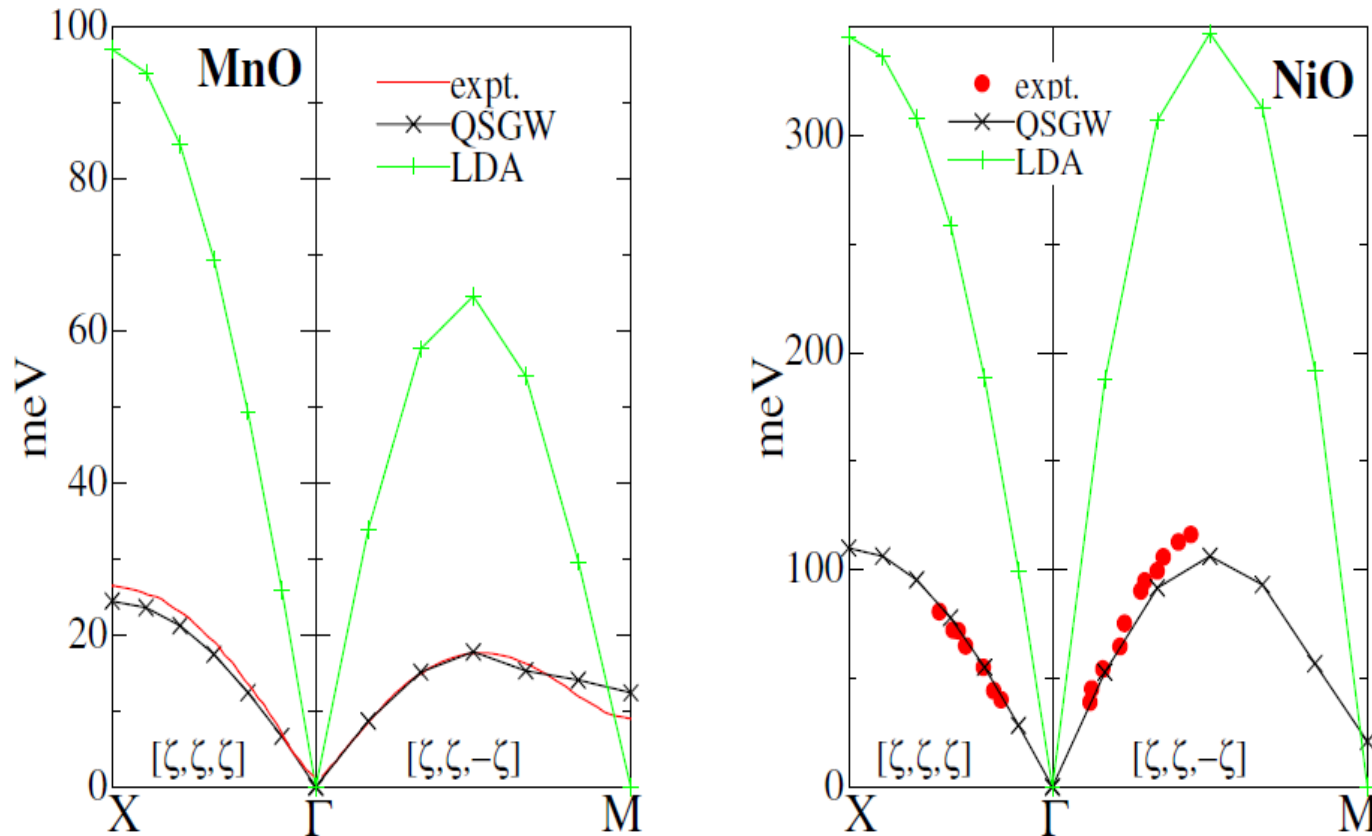
“No Pushing down 3d-band” in GW.



Black:QSGW Red:LDA Blue: e-only



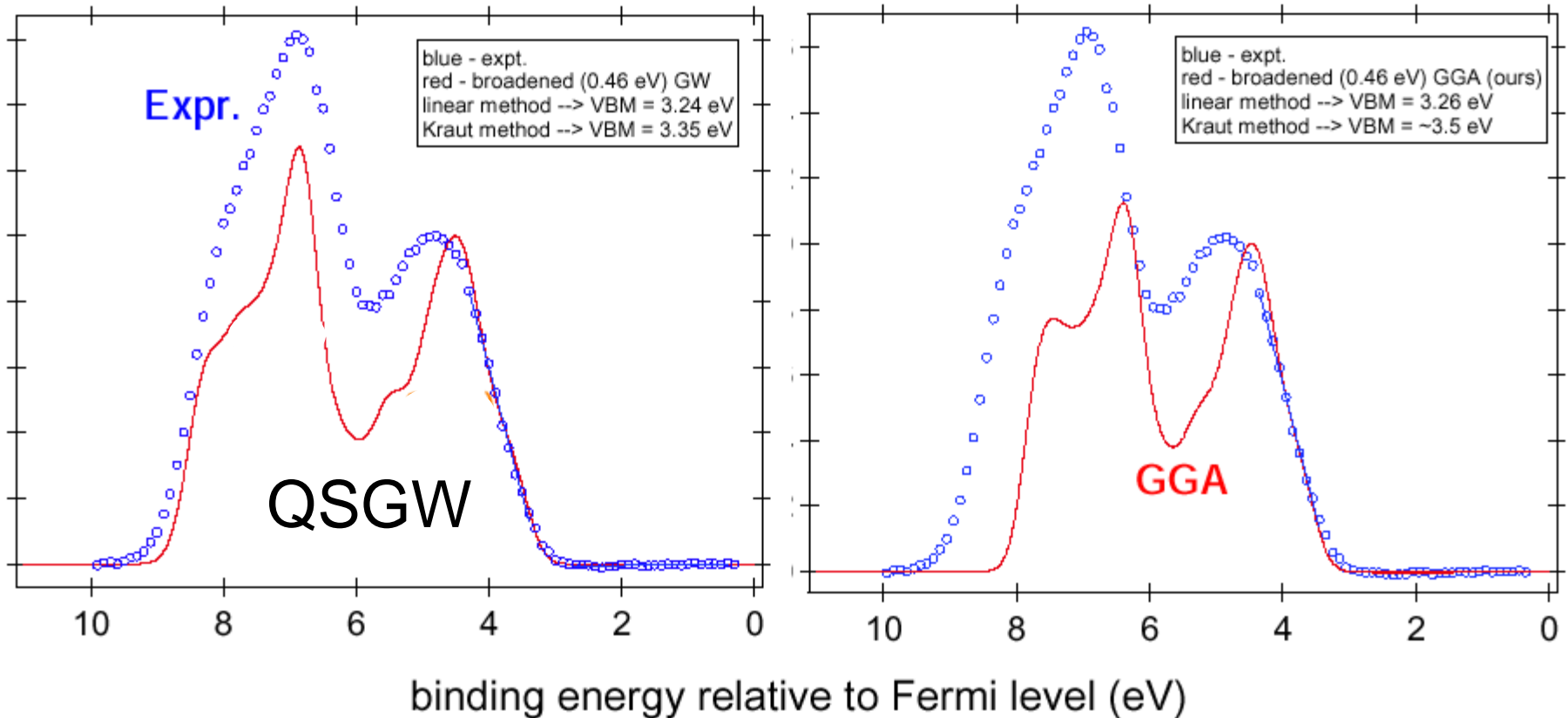
# Spin wave dispersion based on QSGW



J.Phys.C20 (2008) 295214,  
PRB83, 060404(R) (2011) for  $\text{CaFe}_2\text{As}_2$  for spectrum of  $\chi^{+-}$ .

# SrTiO3 Valence DOS

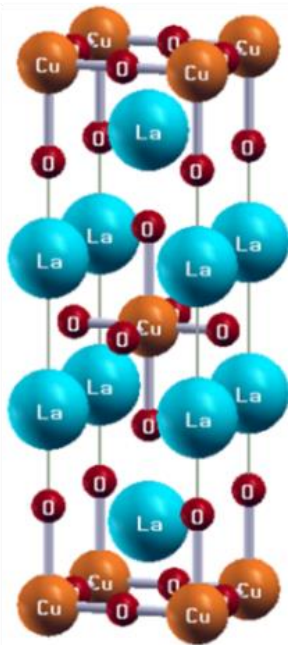
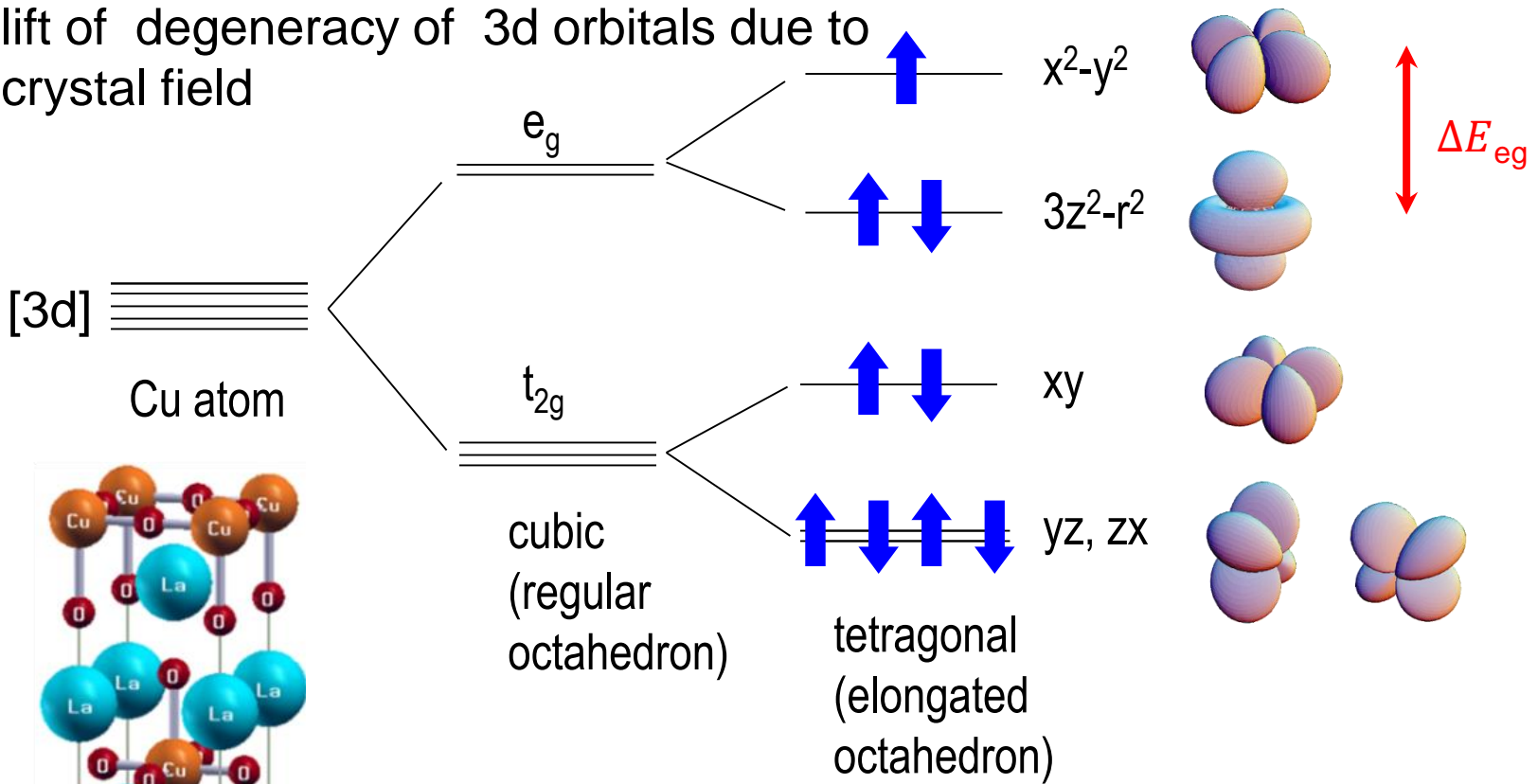
fit of properly broadened theoretical DOS with experiment  
n-STO(001) VB excited with monochromatic AlK $\alpha$  x-rays (resolution = 0.46 eV)



S. A. Chambers et al, Surface Sci 554,81-89 (2004)

# Electron configuration

lift of degeneracy of 3d orbitals due to crystal field



Around  $E_f$ , we have two  $e_g$  bands.

# Band structure:

# Band width and $\Delta E_{eg}$

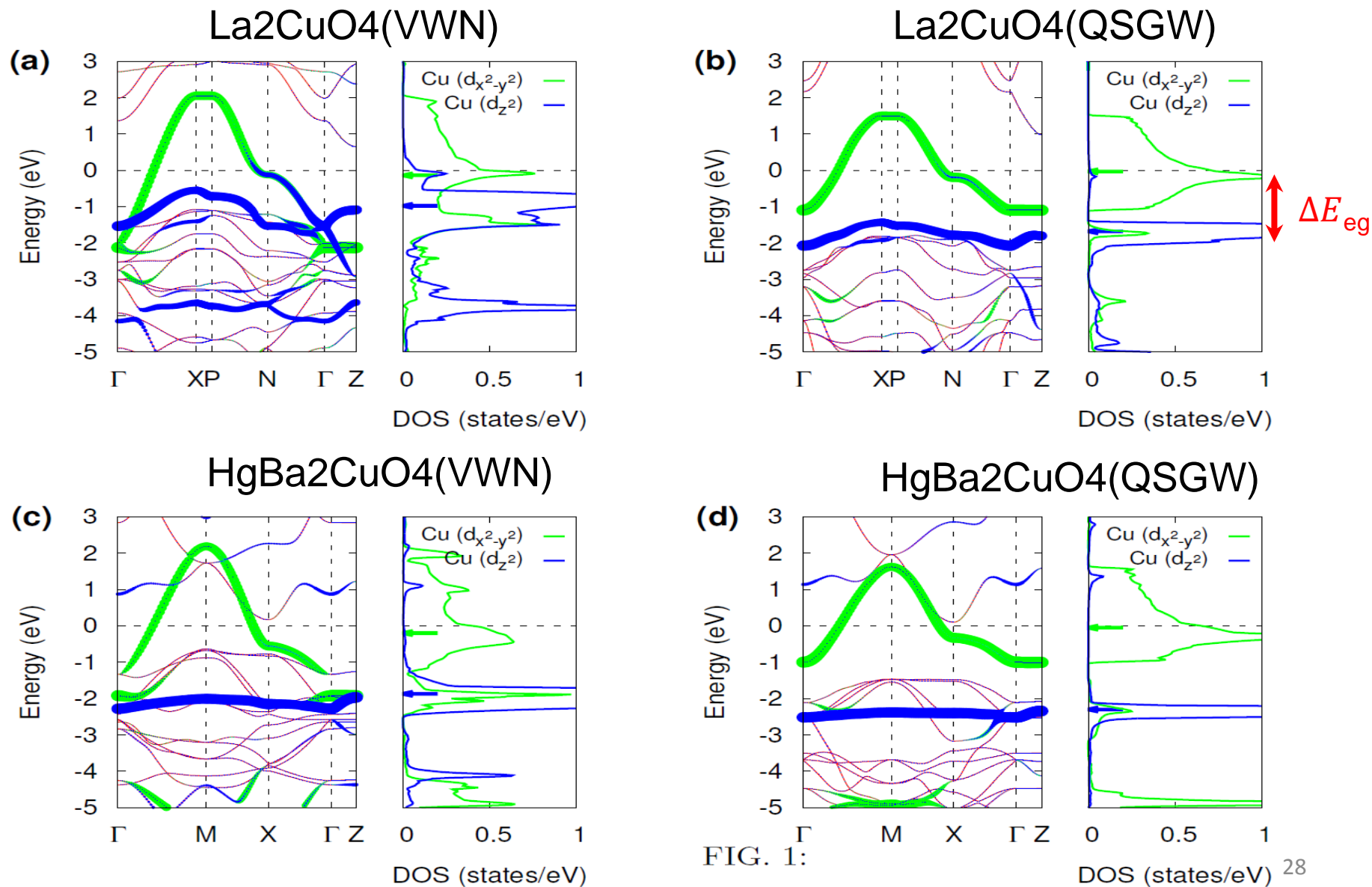
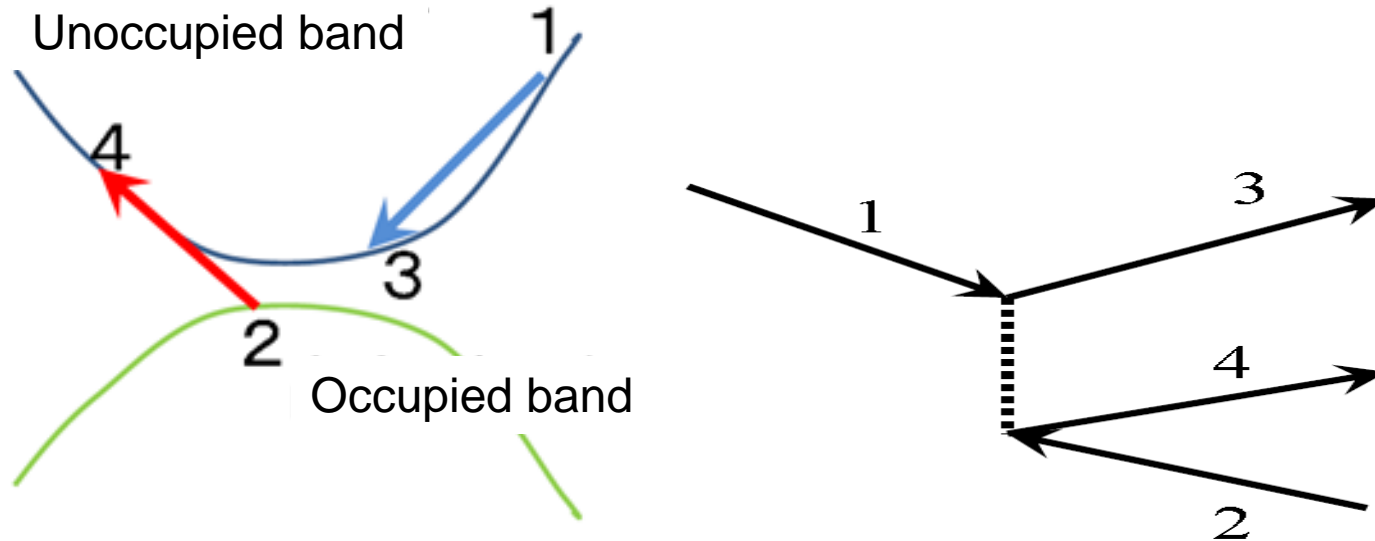


FIG. 1:

# Impact ionization rate(auger process)



Fermi's Golden rule  $\rightarrow$  transition rate. Sum up for all final states. Matrix element of transition is

$$\langle 1,3 | \text{electron-electron interaction} | 4,2 \rangle$$

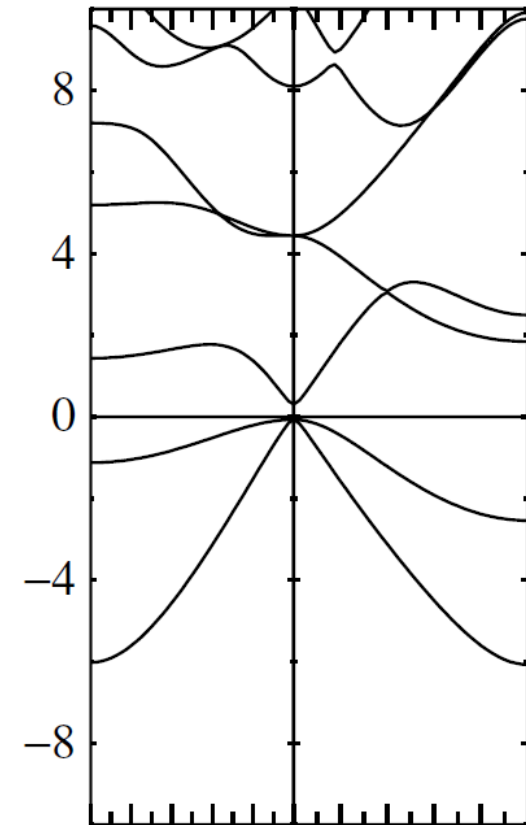
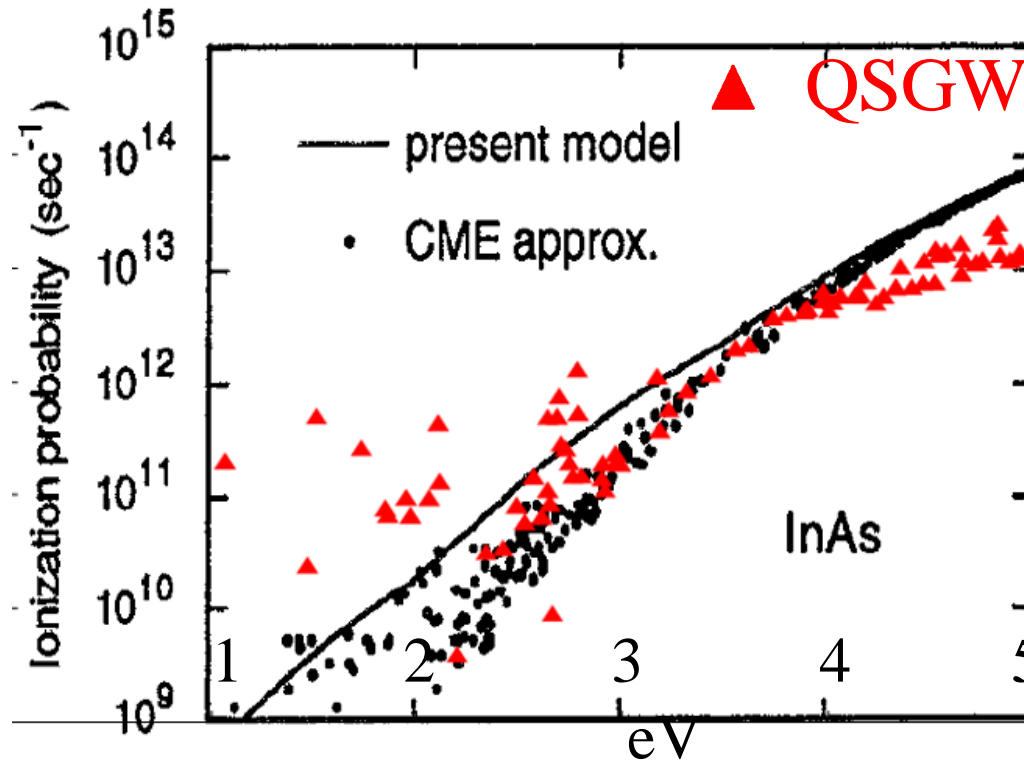
(Energy conservation and Momentum conservation)

# InAs impact ionization rate

(=auger process

= high energy electron lose energy with e-h pair)

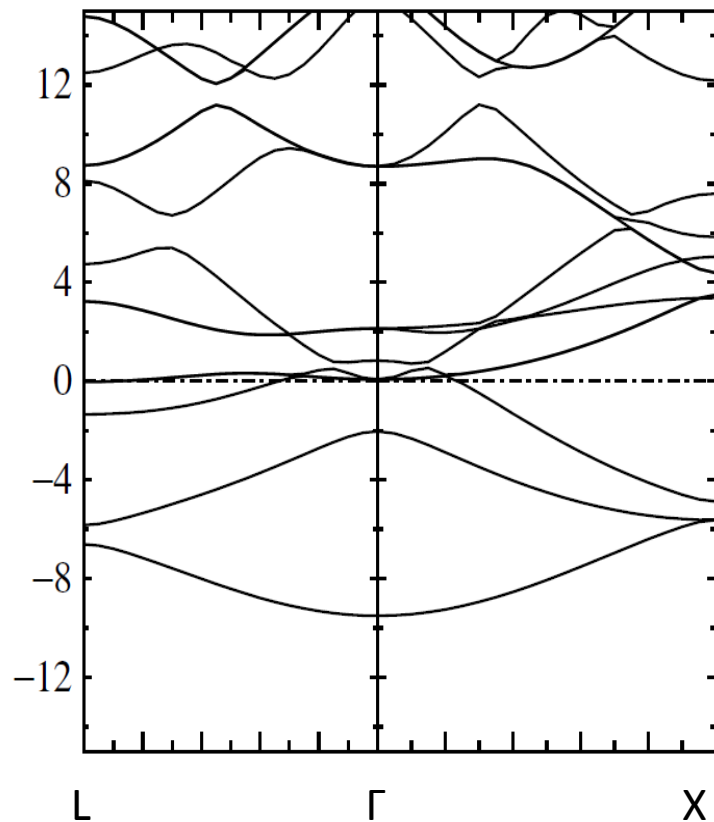
direct gap 0.38eV



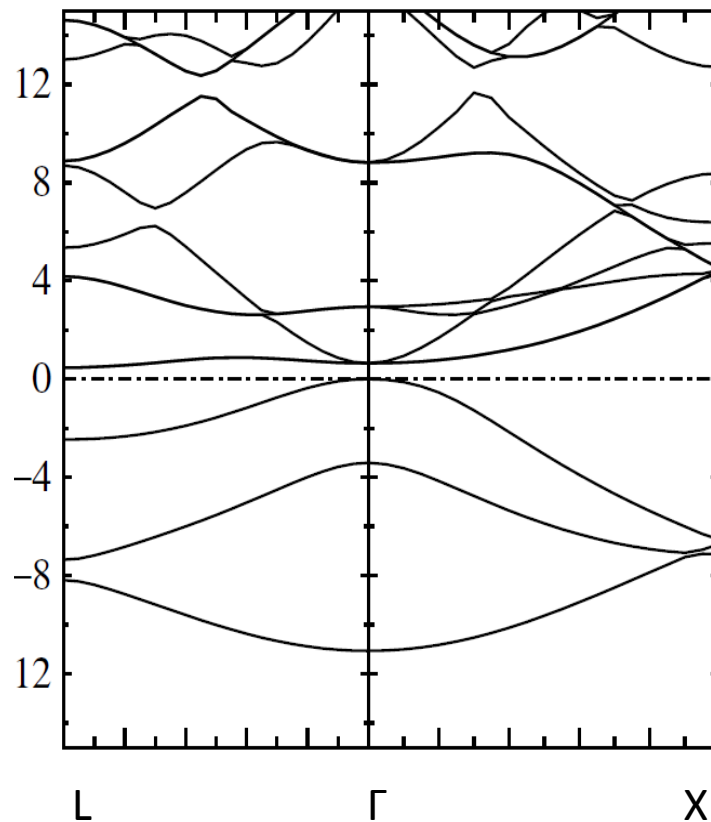
Present model: Sano and Yoshii JAP77 2020 (1995)

# YH3(fcc structure 6x6x6)

## GGA

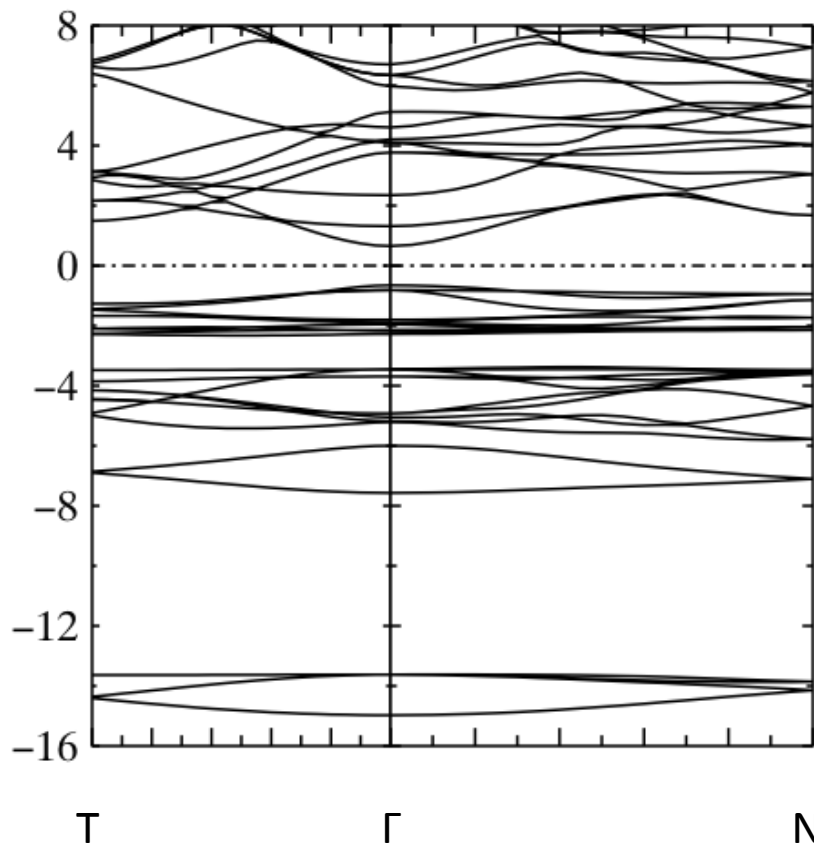


## QSGW

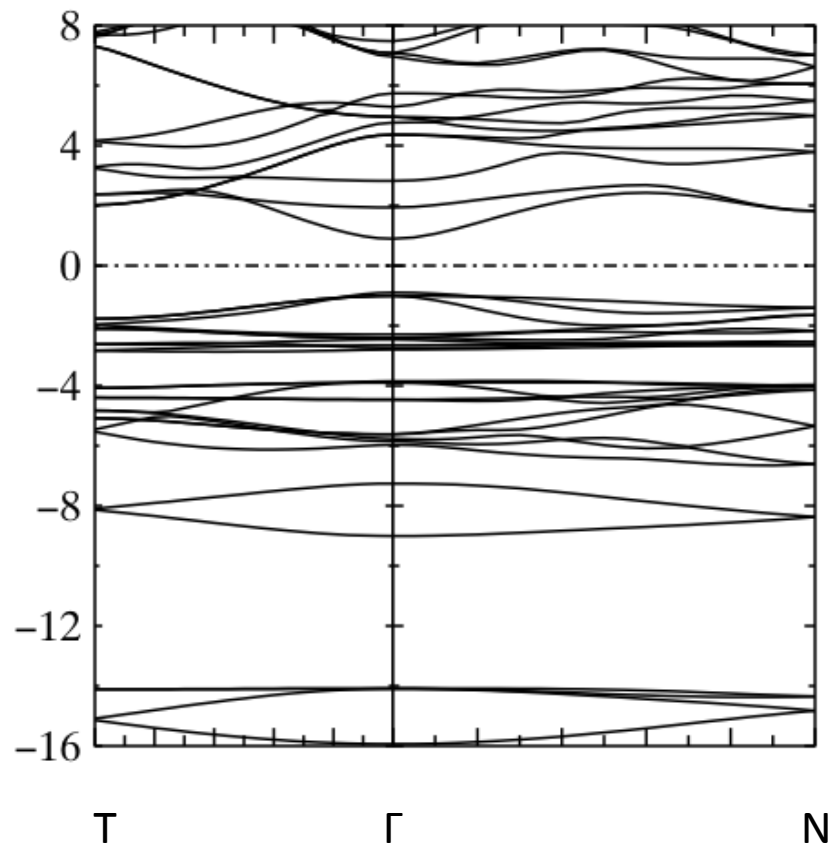


# CuGaSe<sub>2</sub> (eight atoms in cell, 2x2x2. ~3hours per iteration by single core)

## 1shotGW



## QSGW



Band gap GGA:0.4eV, GW(1shot noZ) 1.37 eV,  
QSGW(1.79eV) Exper. 1.63, 1.67, 1.73eV

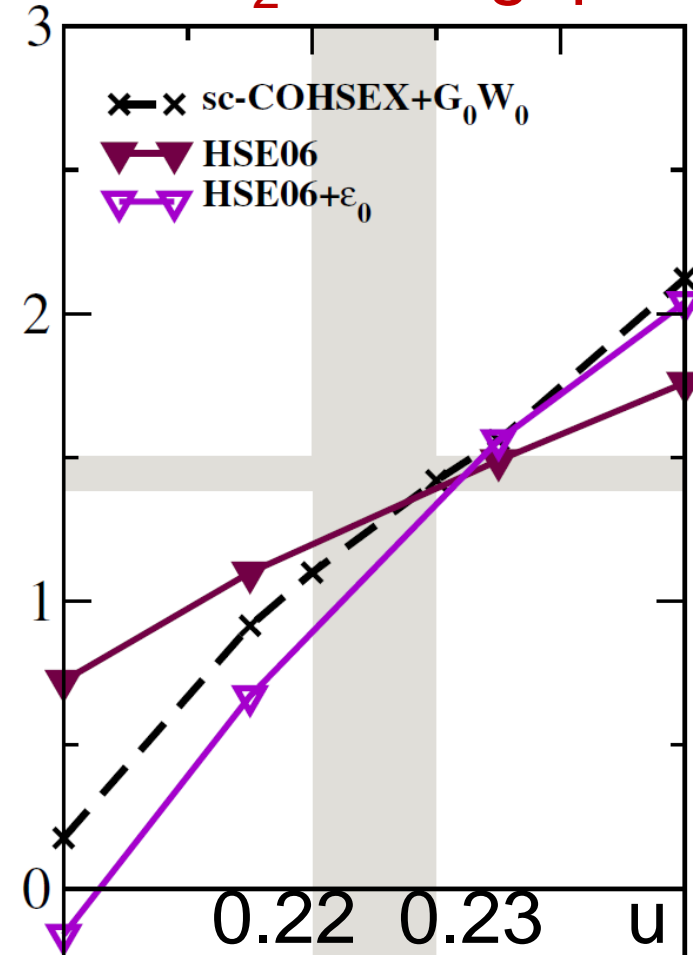


# Band gap for CuAB<sub>2</sub> (eV)

B \ A	S	Se
Al	3.62 (3.49)	2.91 (2.67)
Ga	2.83 (2.43)	1.69 (1.68)
In	1.22 <sub>(u=0.22)</sub> 1.48 <sub>(u=0.23)</sub> (1.53)	----- (1.04)

Experiments in (...)

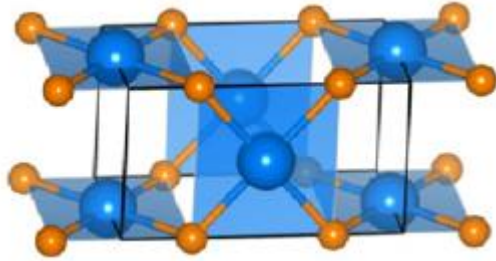
# CuInS<sub>2</sub> band gap (eV)



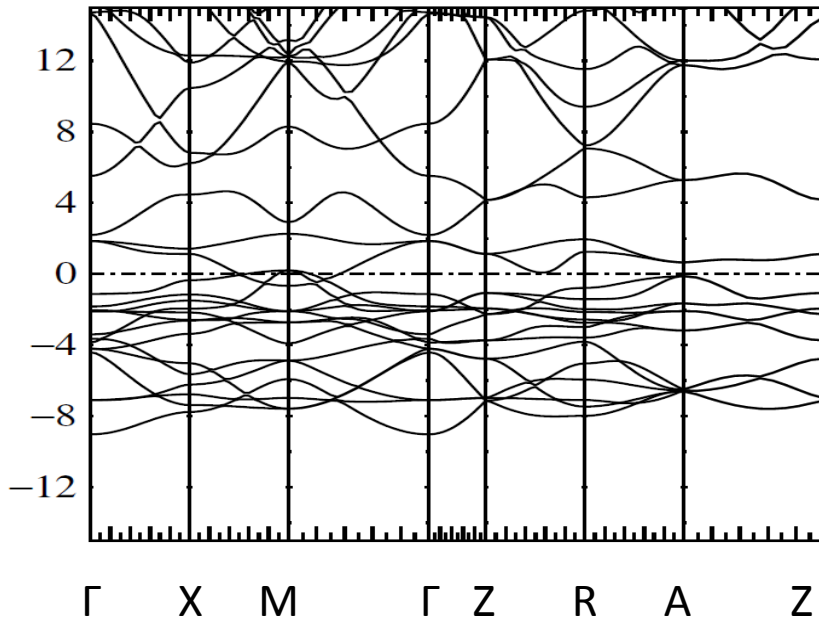
J.Vidal, PRL 104, 056401 (2010)

3x3x3 calculation(need check)

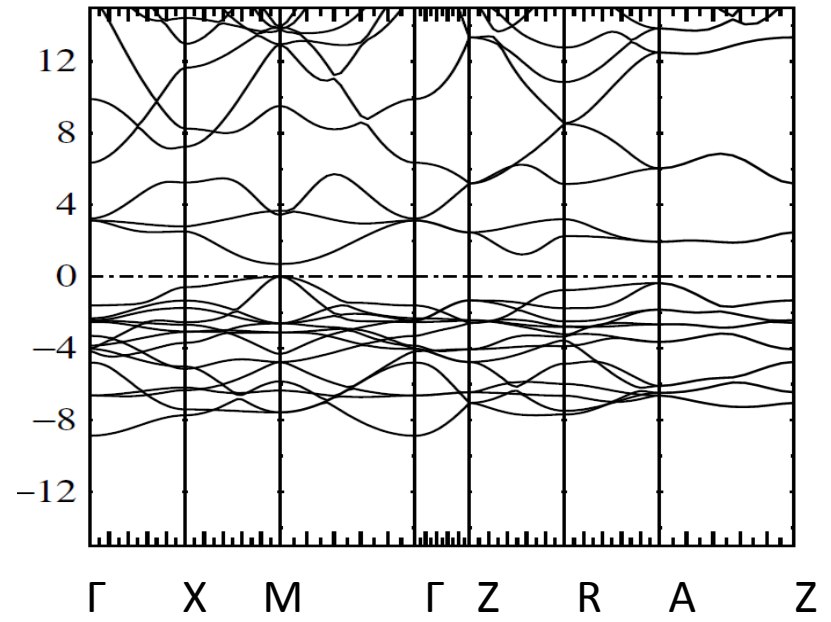
**PdO**(tetragonal , four atoms per cell 3x3x2 calculation , 1hour per iteration )



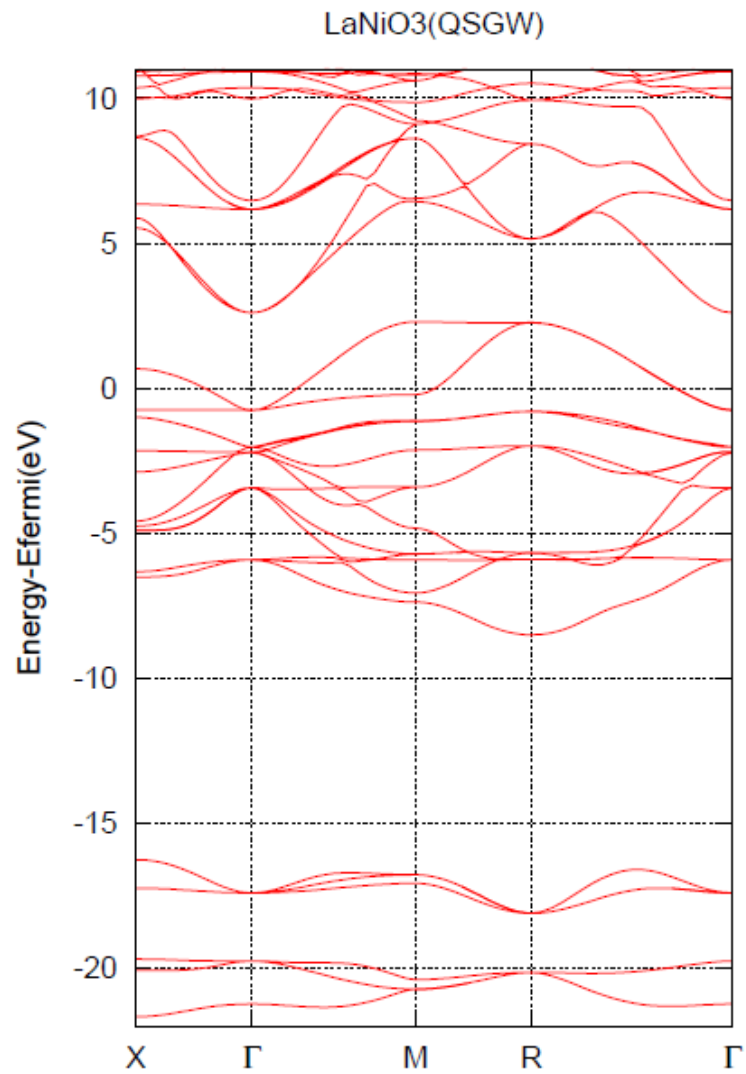
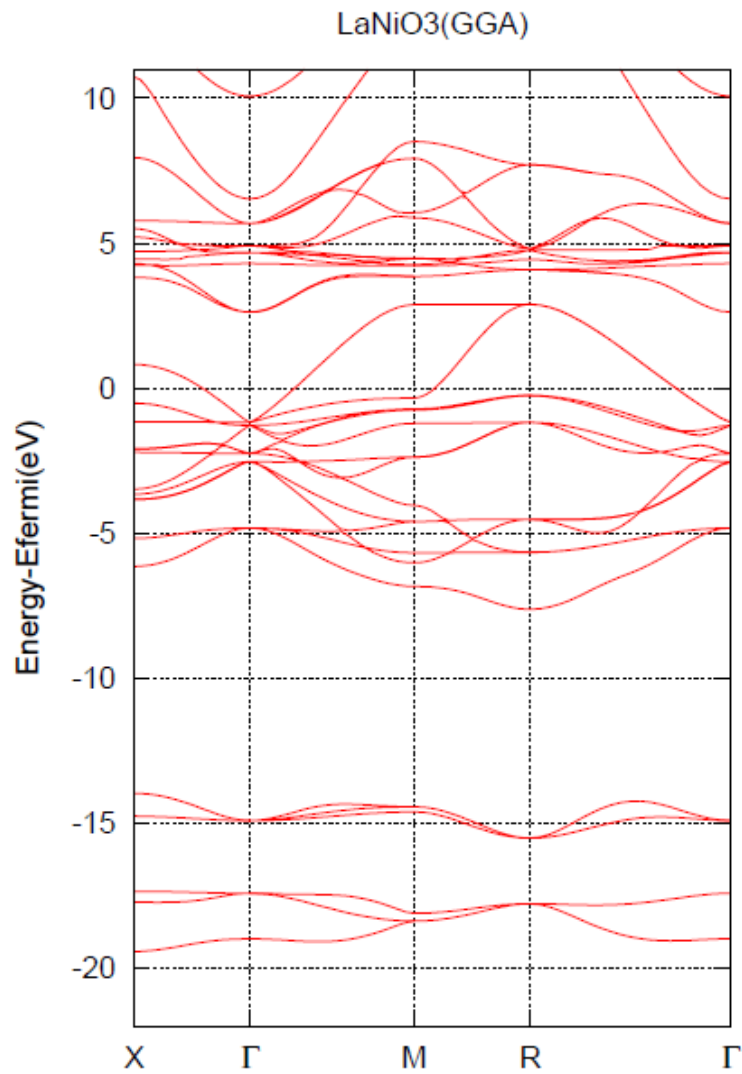
**GGA**



**QSGW**



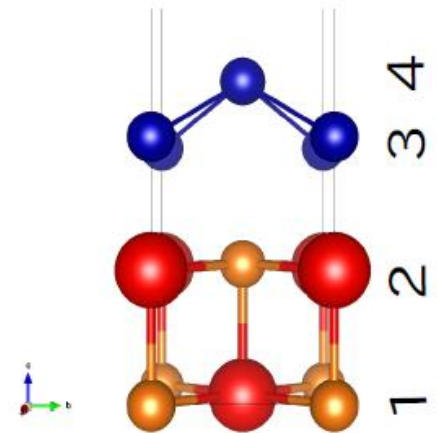
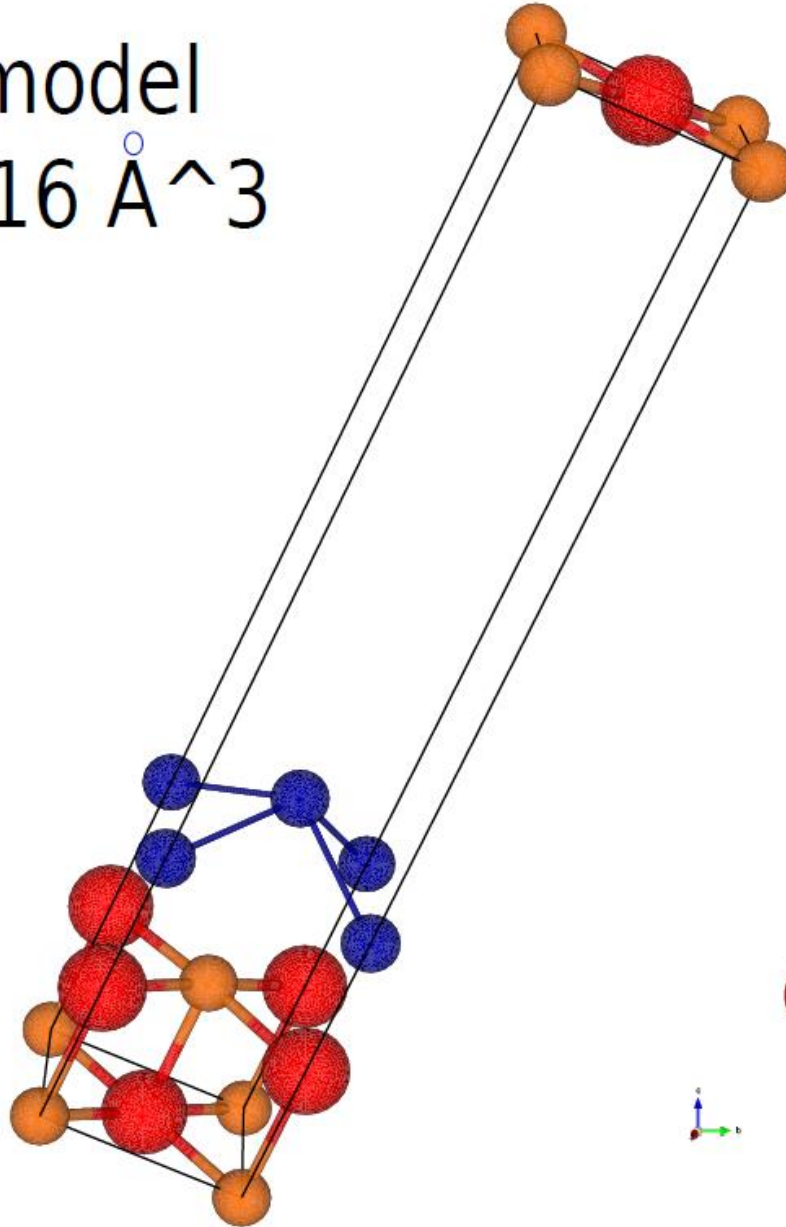
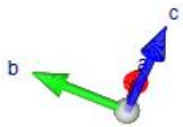
Band gap about 0.7 eV

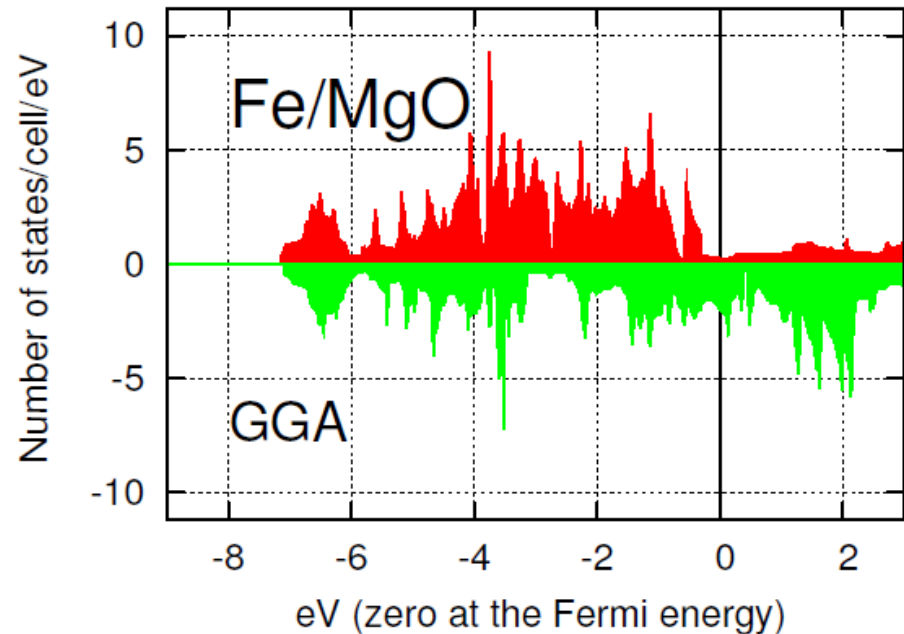
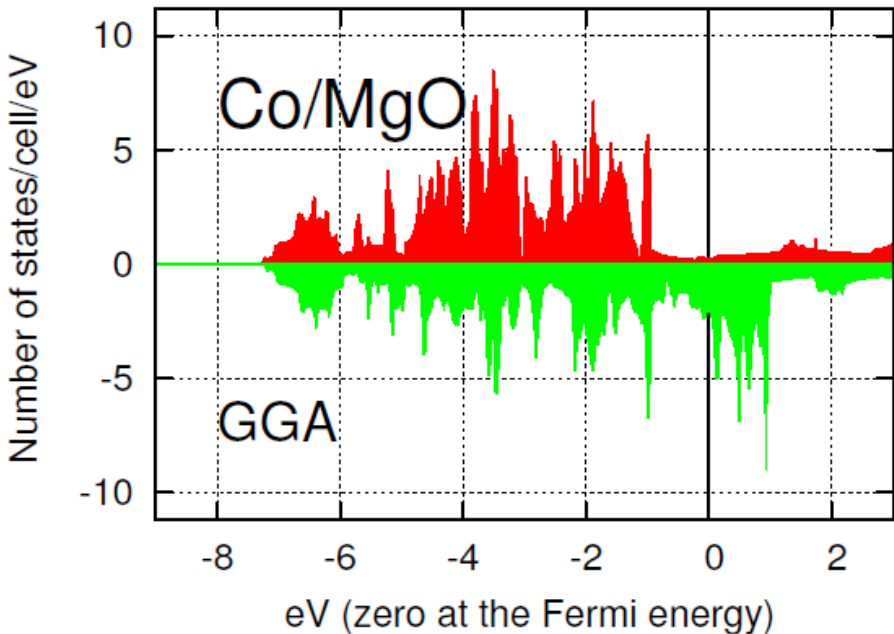
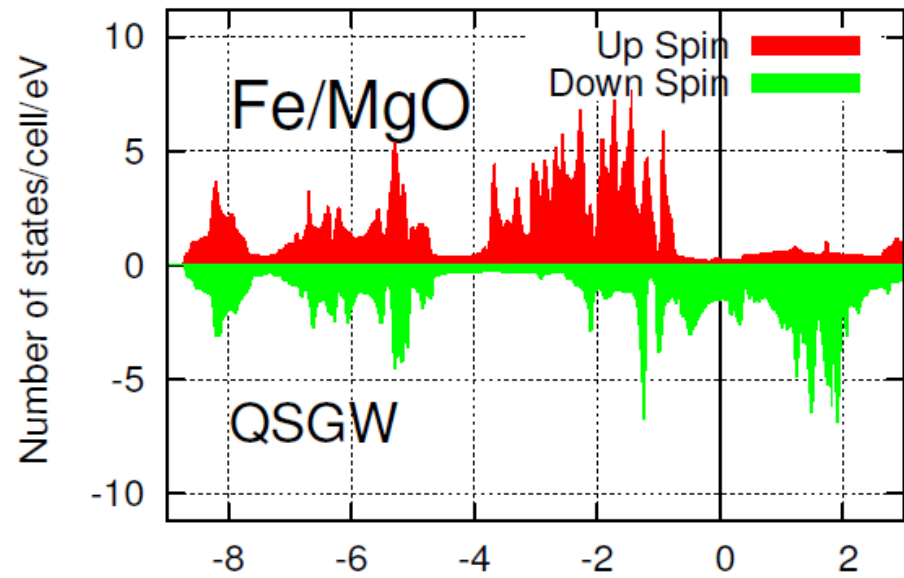
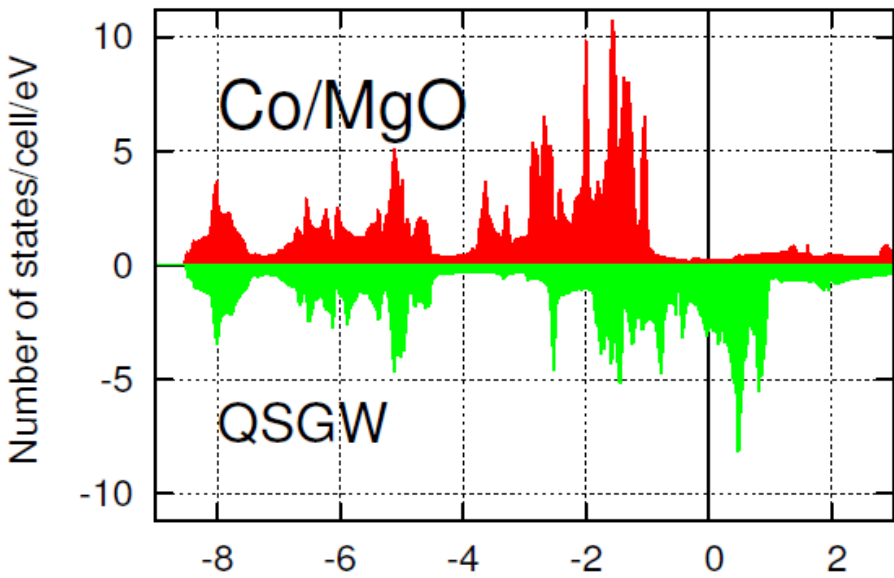


LaNiO<sub>3</sub>

M.J.Han, H.Kino, T.K (arXiv:1402.4915)

Co/MgO model  
cell  $3 \times 3 \times 16 \text{ \AA}^3$





# summary

- Mean field theory and excitation energy  
LDA, Hartree-Fock and QSGW  
Janak theorem, finite vs.infinite system
- RPA total energy  
QSGW method
- Some Results  
Band gap, GaAs and Na, Cu  
NiO, Spin Wave, ZnO, LaMnO<sub>3</sub>,  
InAs (impact ionization), YH<sub>3</sub>, CuGaSe<sub>2</sub>,PdO
- How QSGW works for atoms and molecules?  
See F.Bruneval J.Chem.Phys 136,194107(2012)